

# Summary of Aarhus workshop

24 – 28 October 2005

Jørgen Christensen-Dalsgaard

# Issues

- Numerical accuracy
- Physical consistency
- Model differences
- Near-surface effects
- Semiconvection

# Intrinsic numerical accuracy

- Compare models computed with a given code and given parameters
- Vary number of meshpoints
- Vary number of timesteps

# Case 1.1

0.9  $M_{\odot}$ ,  $X_c = 0.35$

$^3\text{He}$  in equilibrium

Test effect of no. of meshpoints:

(  $N = 1200$  ) – (  $N = 600$  )

Line styles:

..... :  $\delta \ln T$

----- :  $\delta \ln p$

- - - - - :  $\delta \ln \rho$

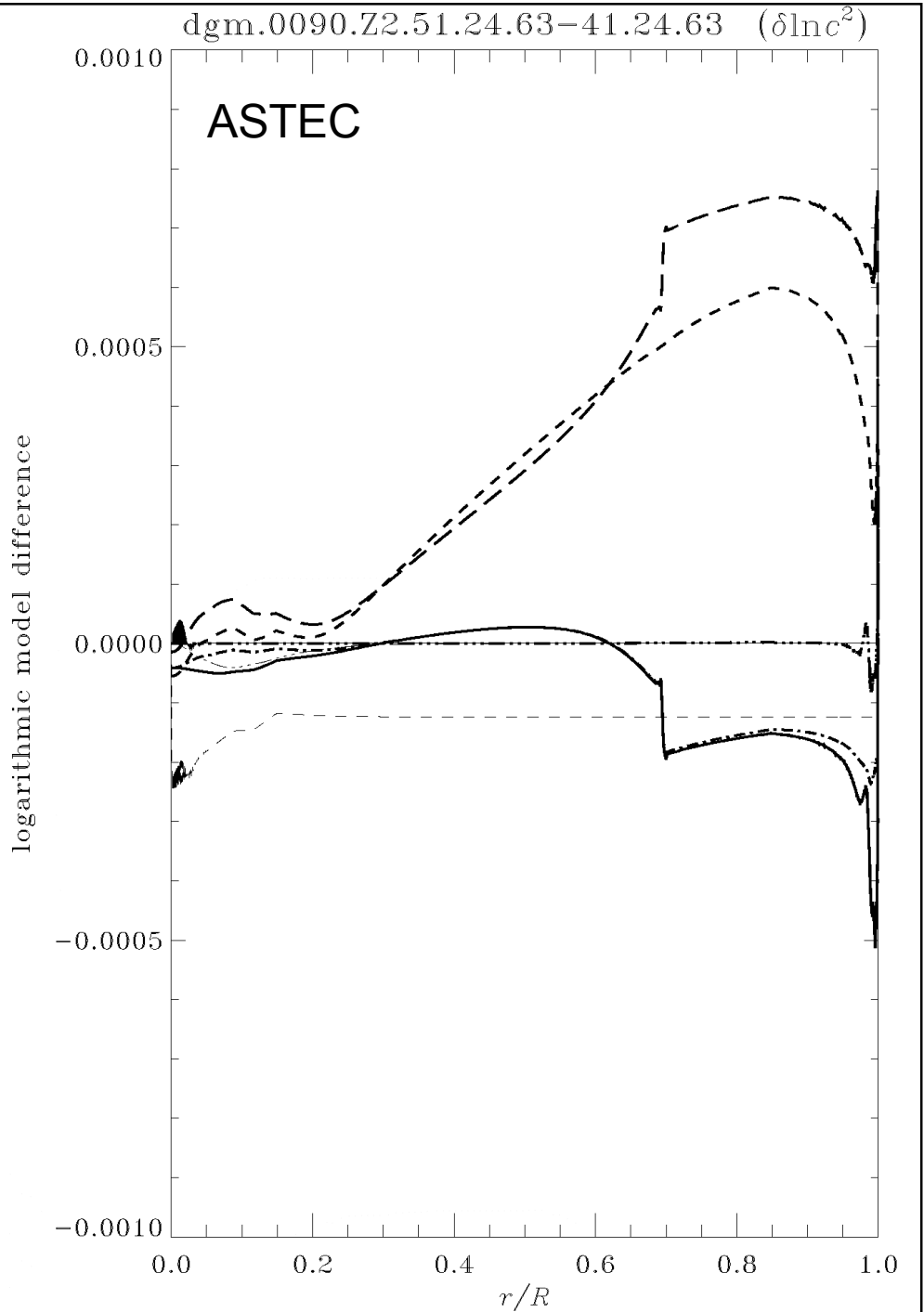
————— :  $\delta \ln c^2$

..... :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$

----- :  $\delta \ln L$

..... :  $\delta X$



# Case 1.1

0.9  $M_{\odot}$ ,  $X_c = 0.35$

$^3\text{He}$  in equilibrium

Test effect of no. timesteps:

( $N_t = 24$ ) - ( $N_t = 13$ )

( $\Delta y_{\max} = 0.025$ ) - ( $\Delta y_{\max} = 0.05$ )

Line styles:

..... :  $\delta \ln T$

----- :  $\delta \ln p$

----- :  $\delta \ln \rho$

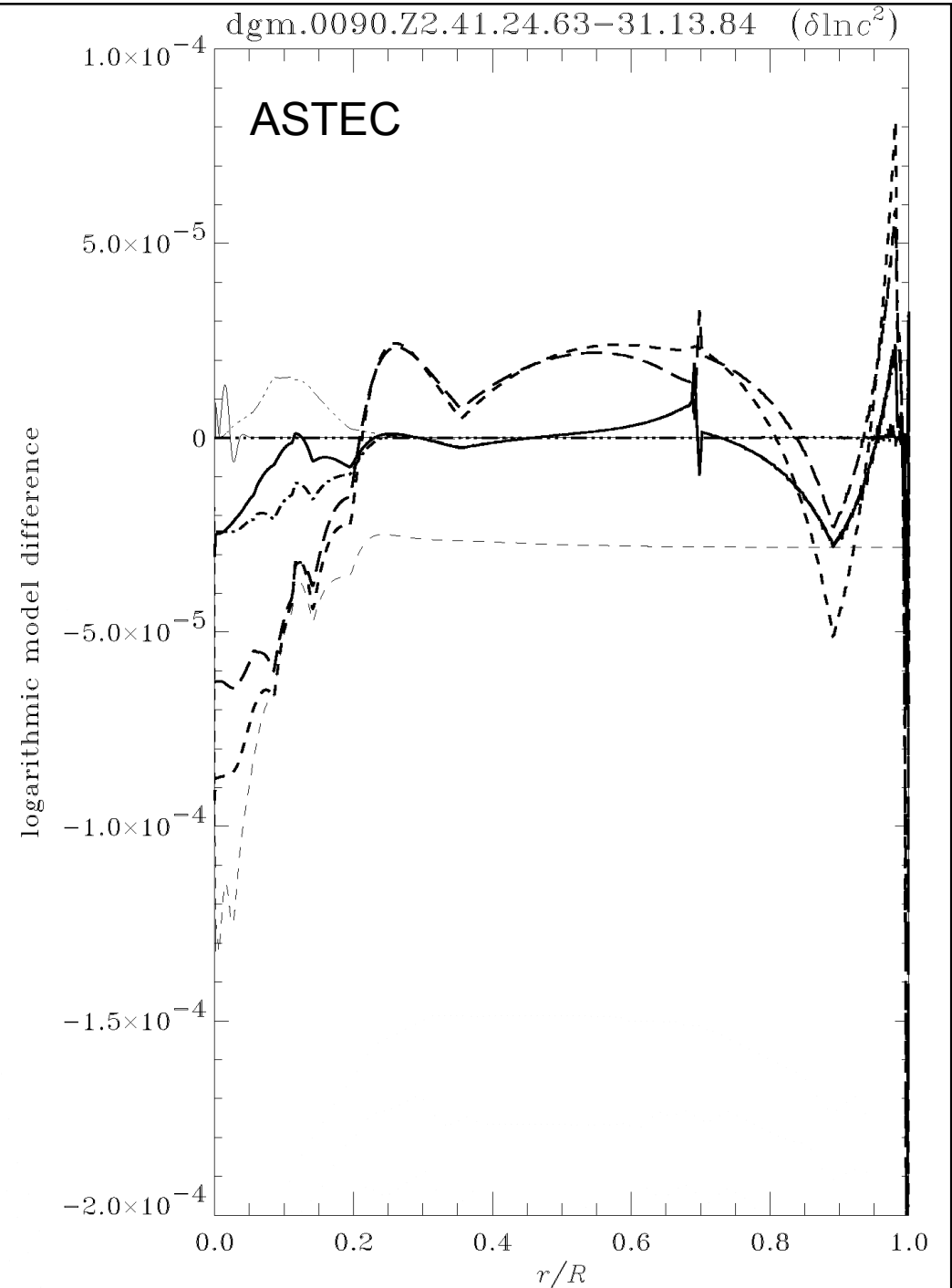
———— :  $\delta \ln c^2$

..... :  $\delta \ln \Gamma_1$

———— :  $\delta \ln q$

----- :  $\delta \ln L$

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# Case 1.3

1.2  $M_{\odot}$ ,  $M_c = 0.1 M_{\odot}$

$^3\text{He}$  in equilibrium

Test effect of no. of meshpoints:

( $N = 600$ ) – ( $N = 1200$ )

Line styles:

..... :  $\delta \ln T$

----- :  $\delta \ln p$

----- :  $\delta \ln \rho$

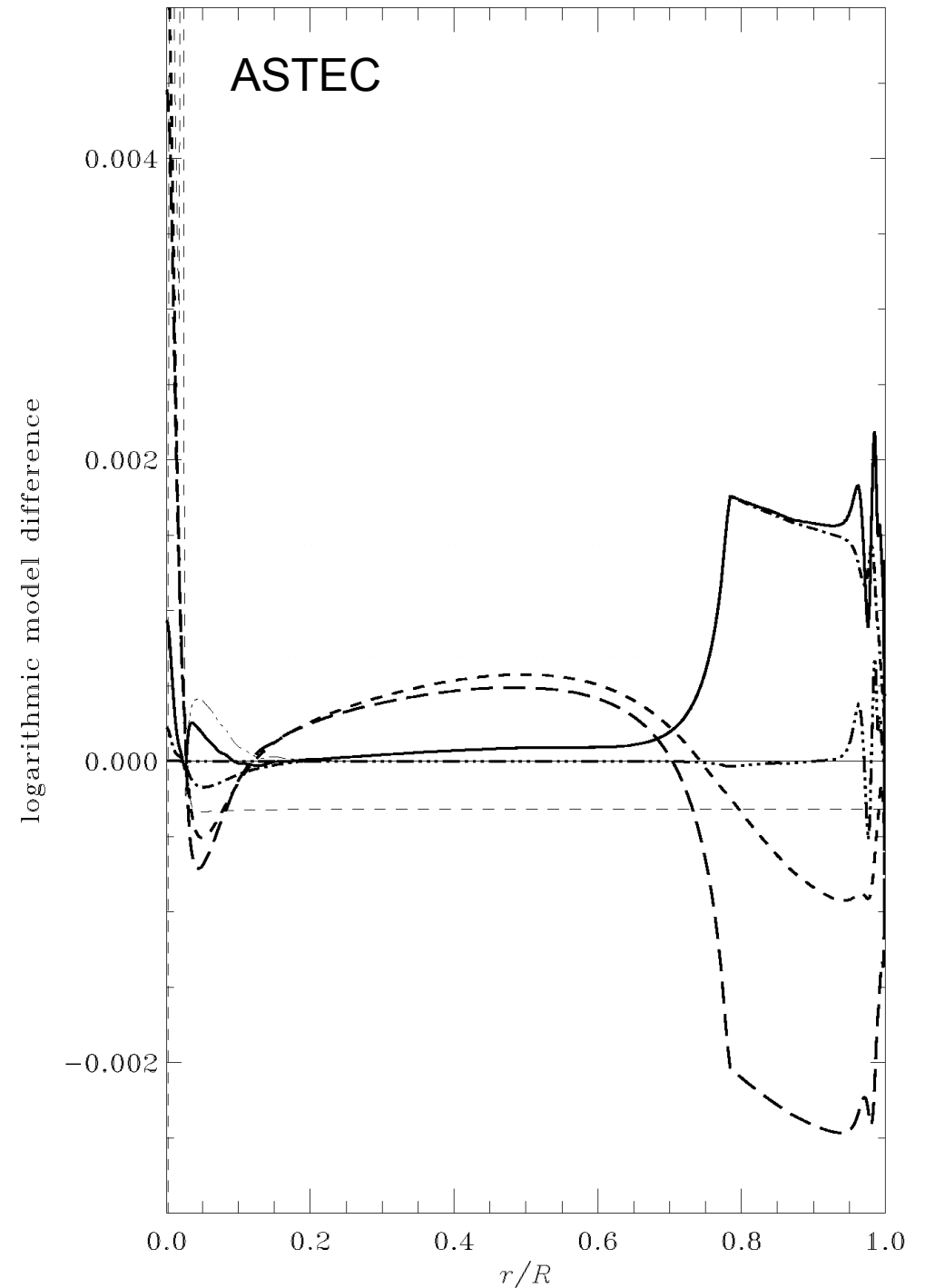
———— :  $\delta \ln c^2$

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———— :  $\delta \ln q$

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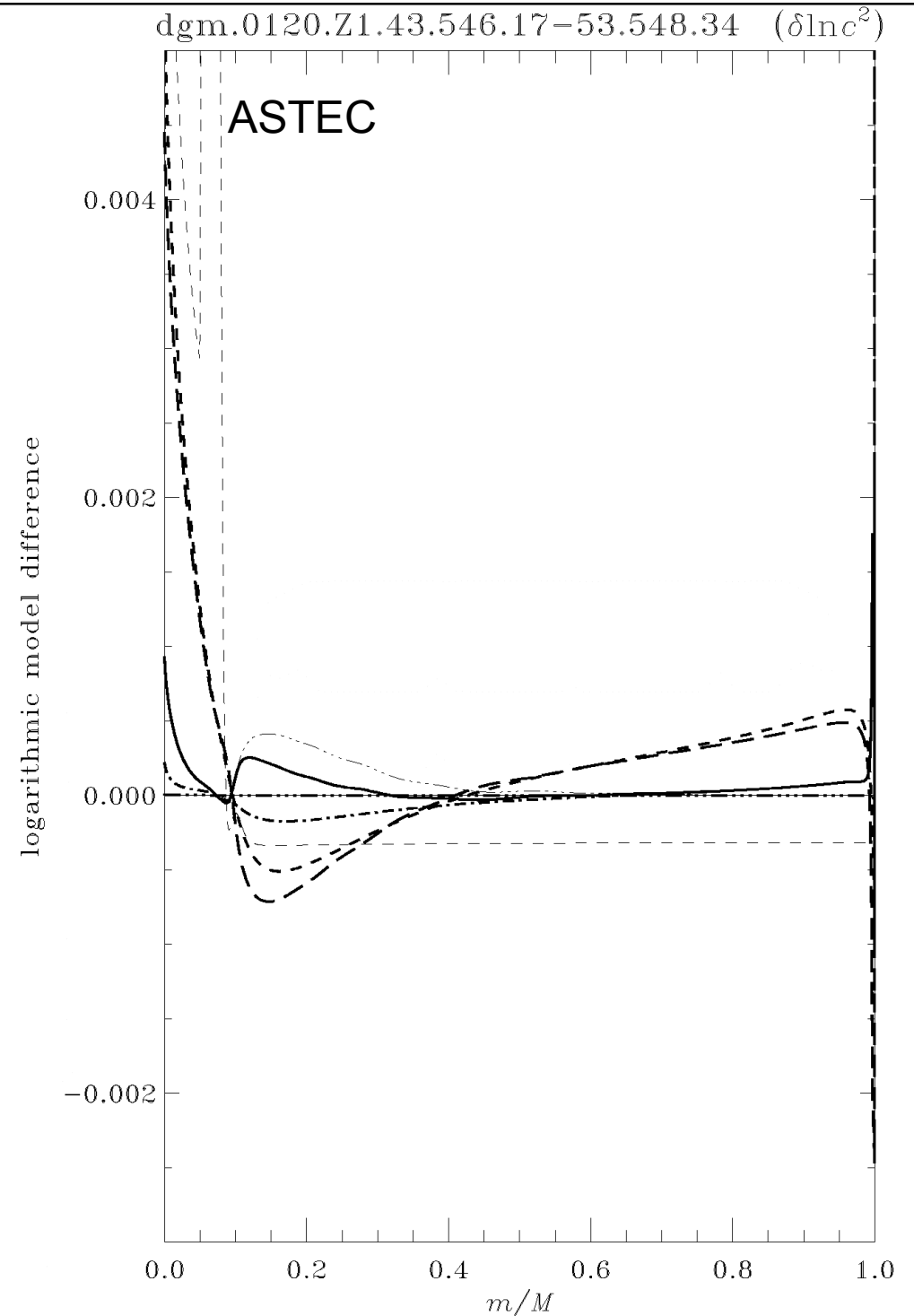
1.2  $M_{\odot}$ ,  $M_c = 0.1 M_{\odot}$

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(  $N = 600$  ) – (  $N = 1200$  )

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- |           |                         |             |                  |
|-----------|-------------------------|-------------|------------------|
| .....     | : $\delta \ln T$        | ————        | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | -----       | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | - · - · - · | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |             |                  |
| .....     | : $\delta \ln \Gamma_1$ |             |                  |



# Case 1.3

1.2 M<sub>⊙</sub>, M<sub>c</sub> = 0.1 M<sub>⊙</sub>  
3He in equilibrium

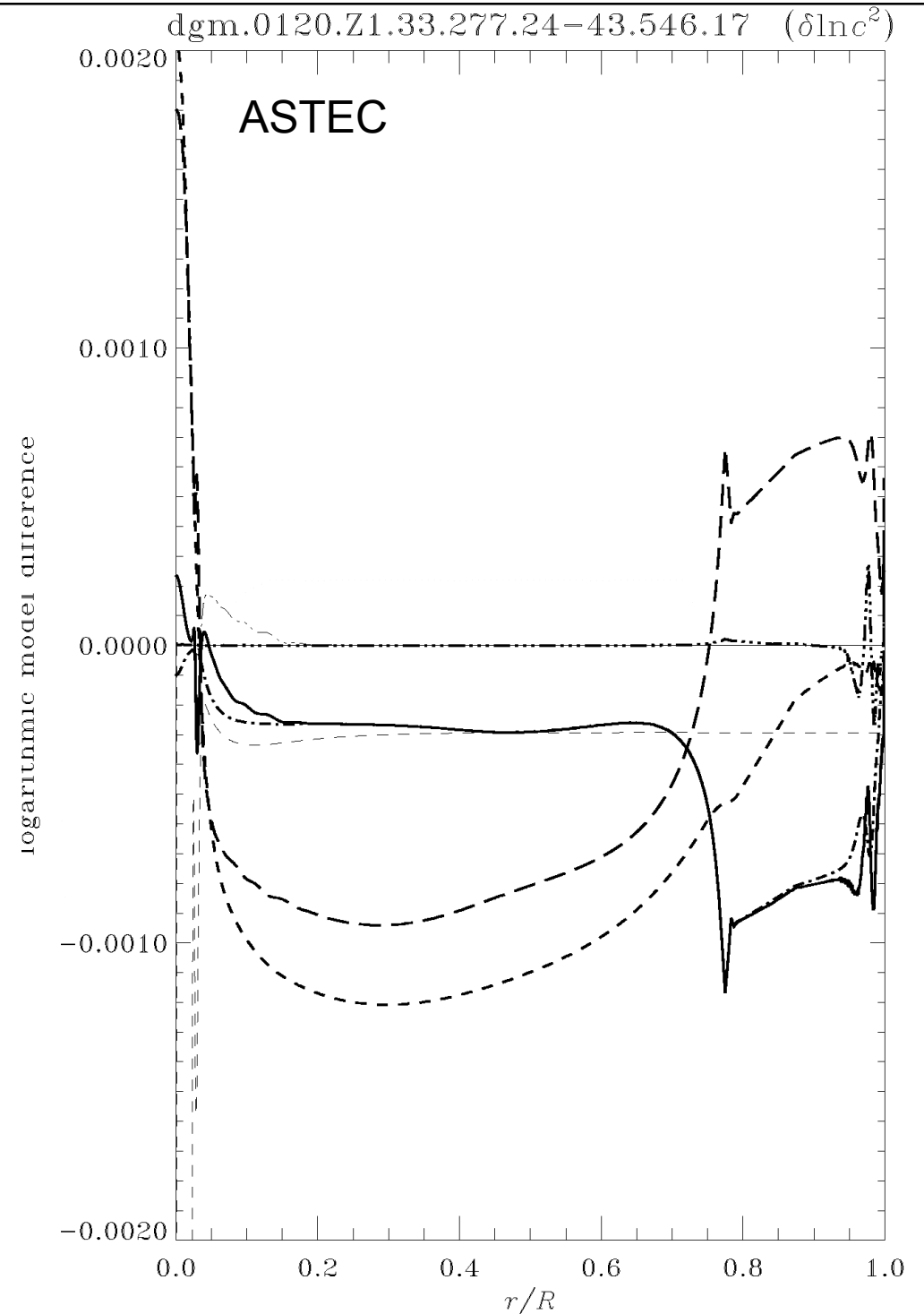
Test effect of no. timesteps:

(N<sub>t</sub> = 277) – (N<sub>t</sub> = 546)

(Δ y<sub>max</sub> = 0.05) – (Δ y<sub>max</sub> = 0.025)

Line styles:

.....	: δlnT	————	: δlnq
-----	: δlnp	-----	: δlnL
- - - - -	: δlnρ	- - - - -	: δX
————	: δln c <sup>2</sup>		
.....	: δlnΓ <sub>1</sub>		





# Case 1.5

2.0  $M_{\odot}$ ,  $X_c = 0.01$ ,

Overshoot 0.15  $H_p$

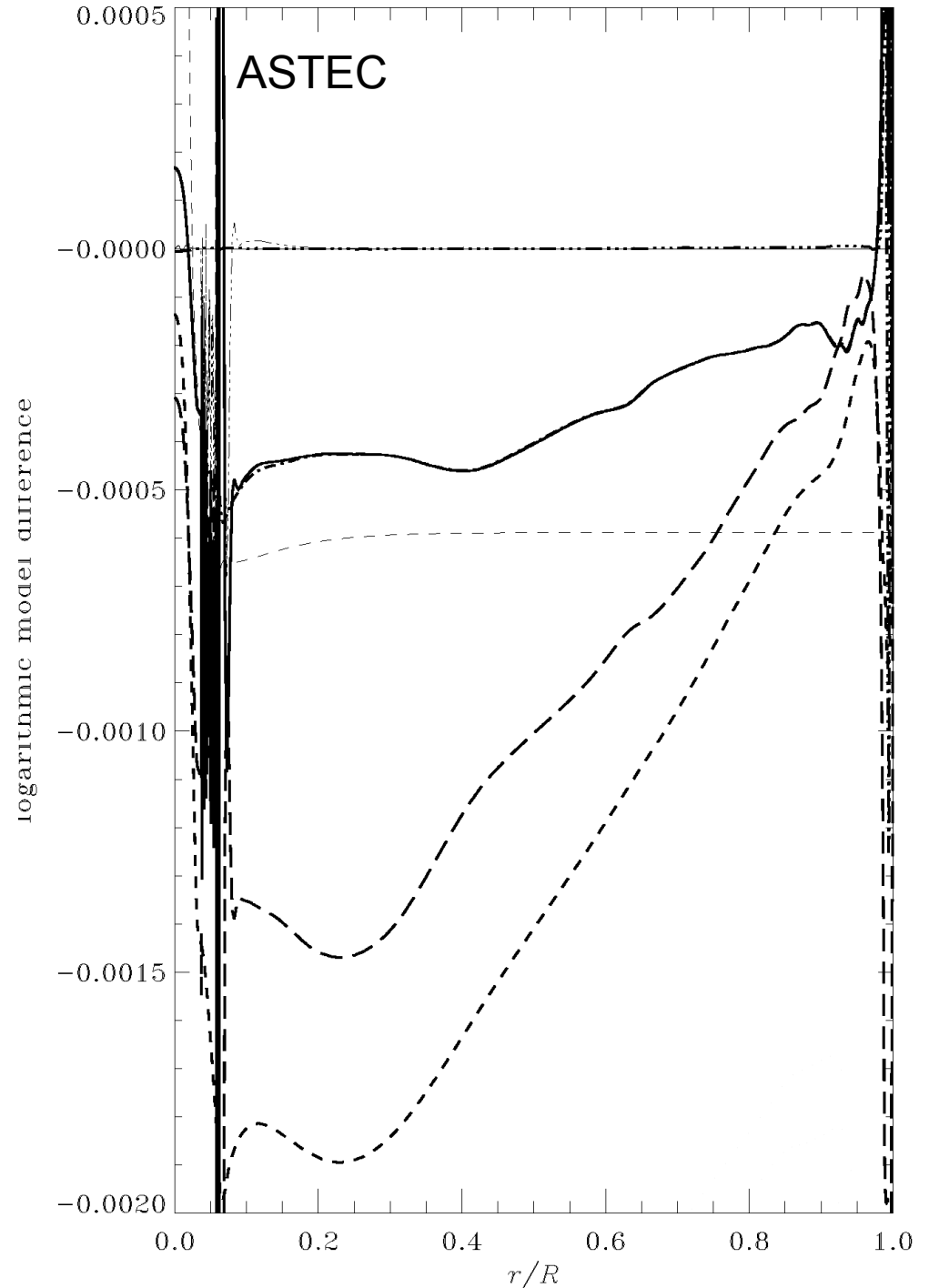
$^3\text{He}$  in equilibrium

Test effect of no. of meshpoints:

(  $N = 600$  ) – (  $N = 1200$  )

Line styles:

- |           |                         |           |                  |
|-----------|-------------------------|-----------|------------------|
| .....     | : $\delta \ln T$        | ————      | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | -----     | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | - - - - - | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |           |                  |
| .....     | : $\delta \ln \Gamma_1$ |           |                  |



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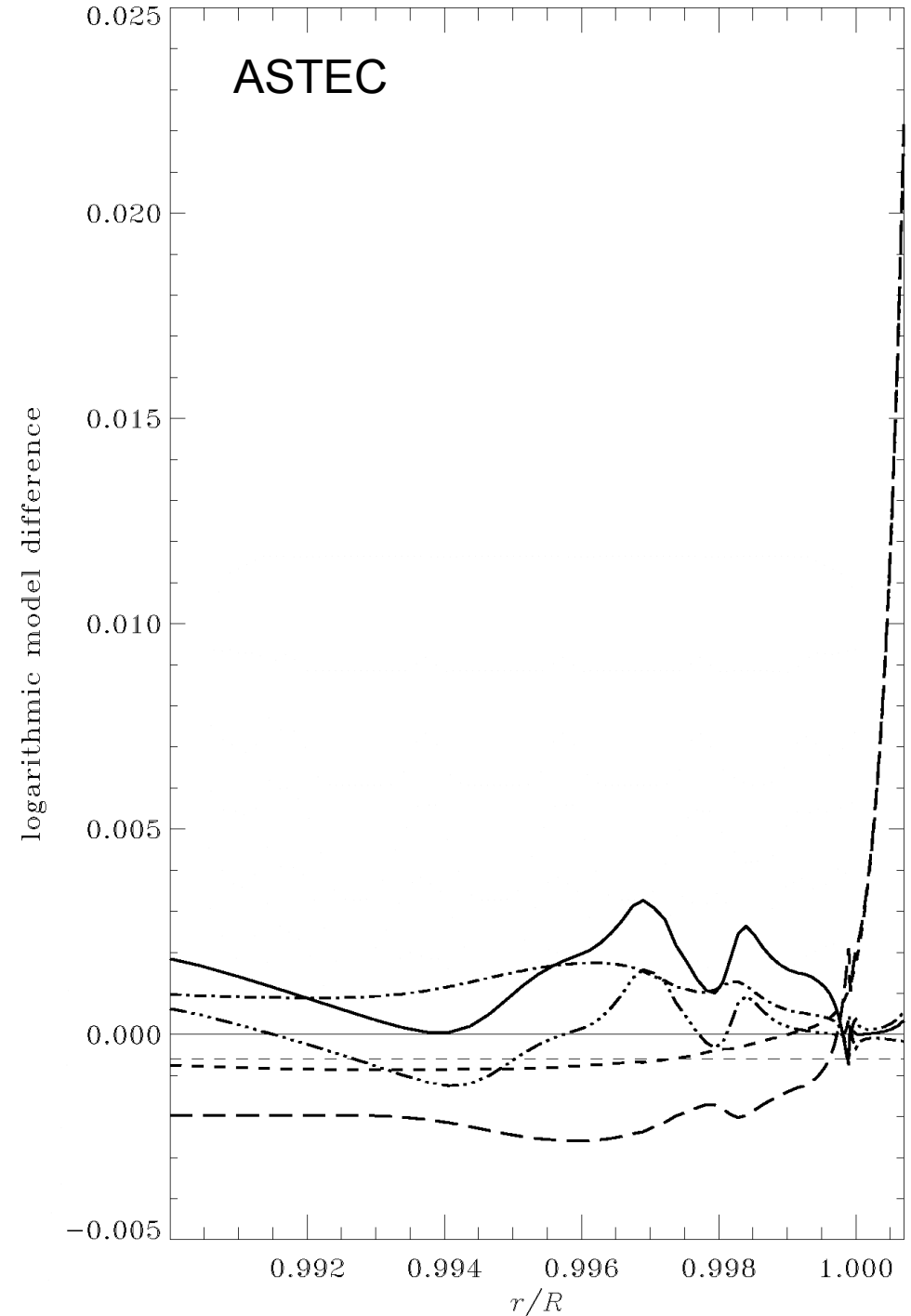
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Overshoot  $0.15 H_p$

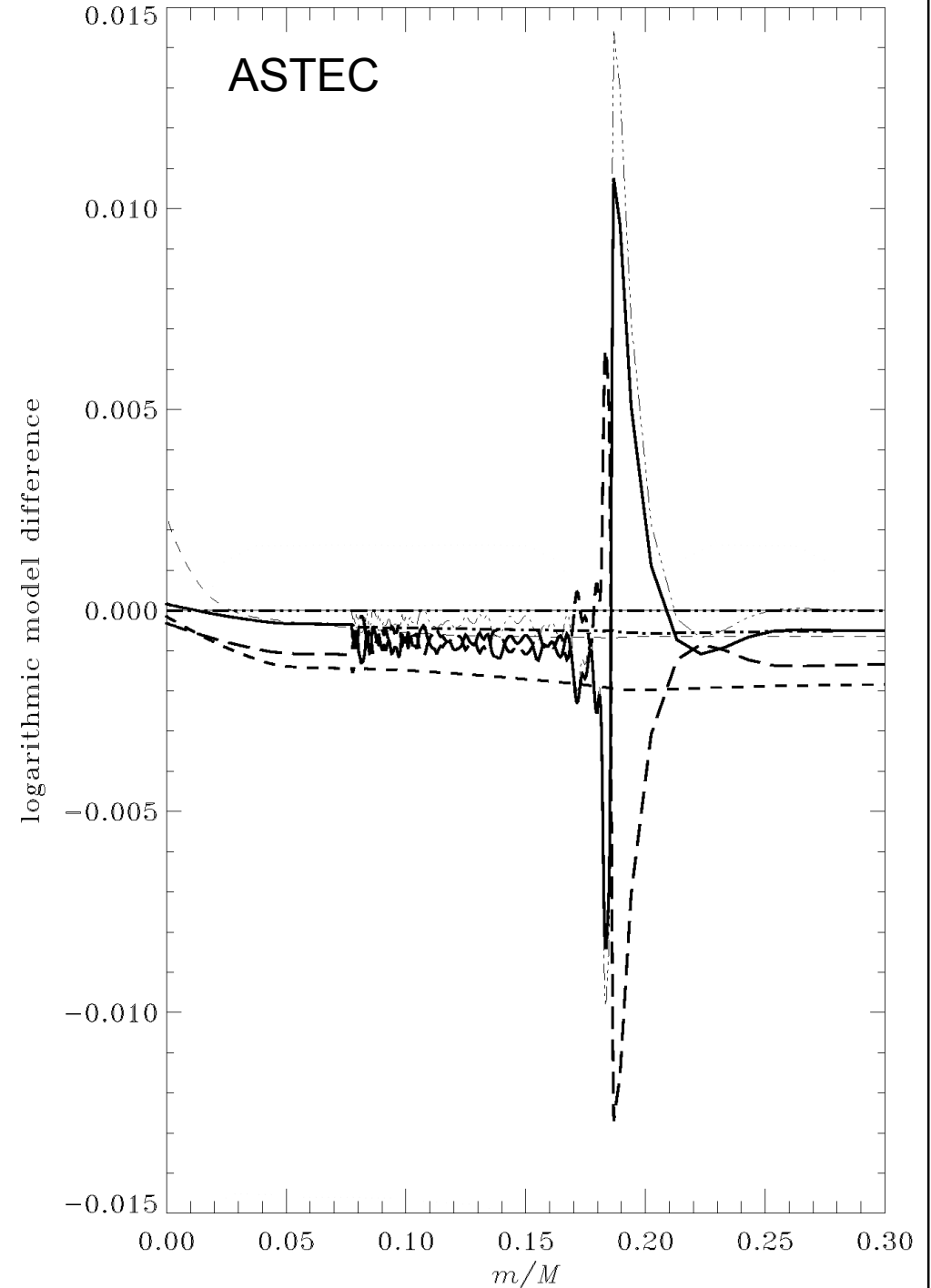
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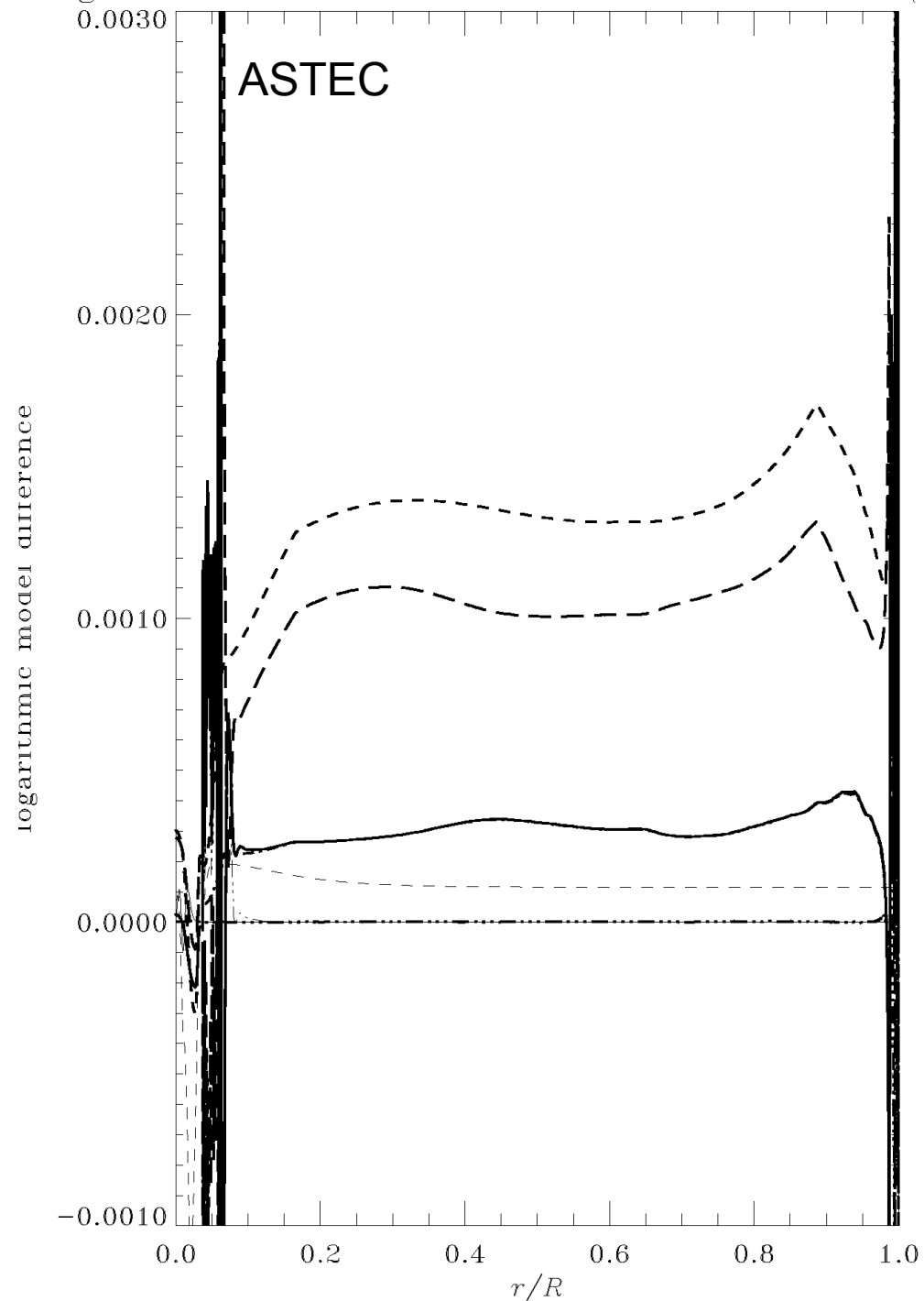
$^3\text{He}$  in equilibrium

Test effect of no. of timesteps:

( $N_t = 208$ ) – ( $N_t = 402$ )

Line styles:

.....	: $\delta \ln T$	————	: $\delta \ln q$
-----	: $\delta \ln p$	-----	: $\delta \ln L$
- - - - -	: $\delta \ln \rho$	- - - - -	: $\delta X$
————	: $\delta \ln c^2$		
.....	: $\delta \ln \Gamma_1$		



# Case 1.5

2.0  $M_{\odot}$ ,  $X_c = 0.01$ ,

Overshoot 0.15  $H_p$

$^3\text{He}$  in equilibrium

Test effect of no. of timesteps:

( $N_t = 208$ ) – ( $N_t = 402$ )

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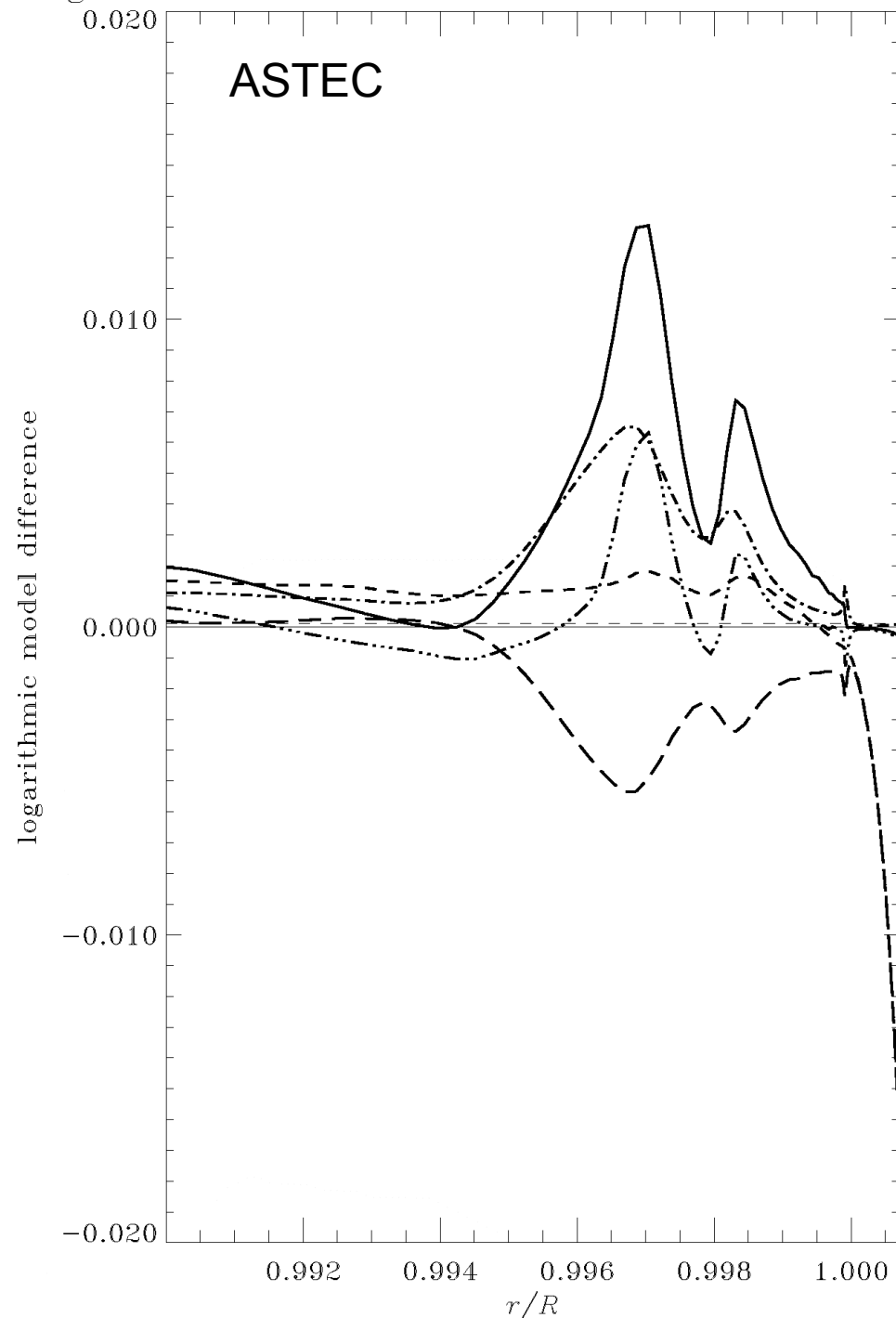
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$2.0 M_{\odot}$ ,  $X_c = 0.01$ ,

Overshoot  $0.15 H_p$

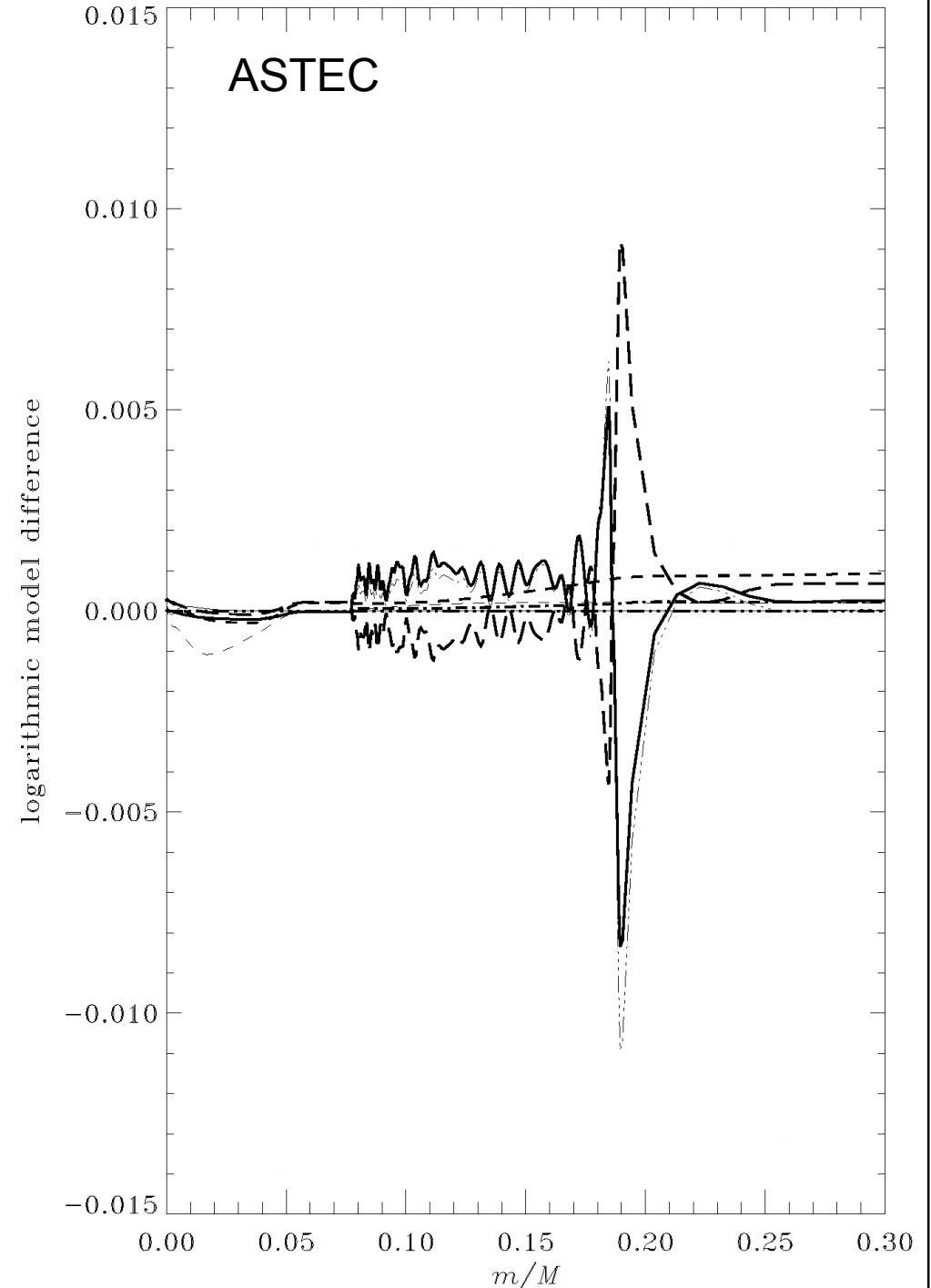
$^3\text{He}$  in equilibrium

Test effect of no. of timesteps:

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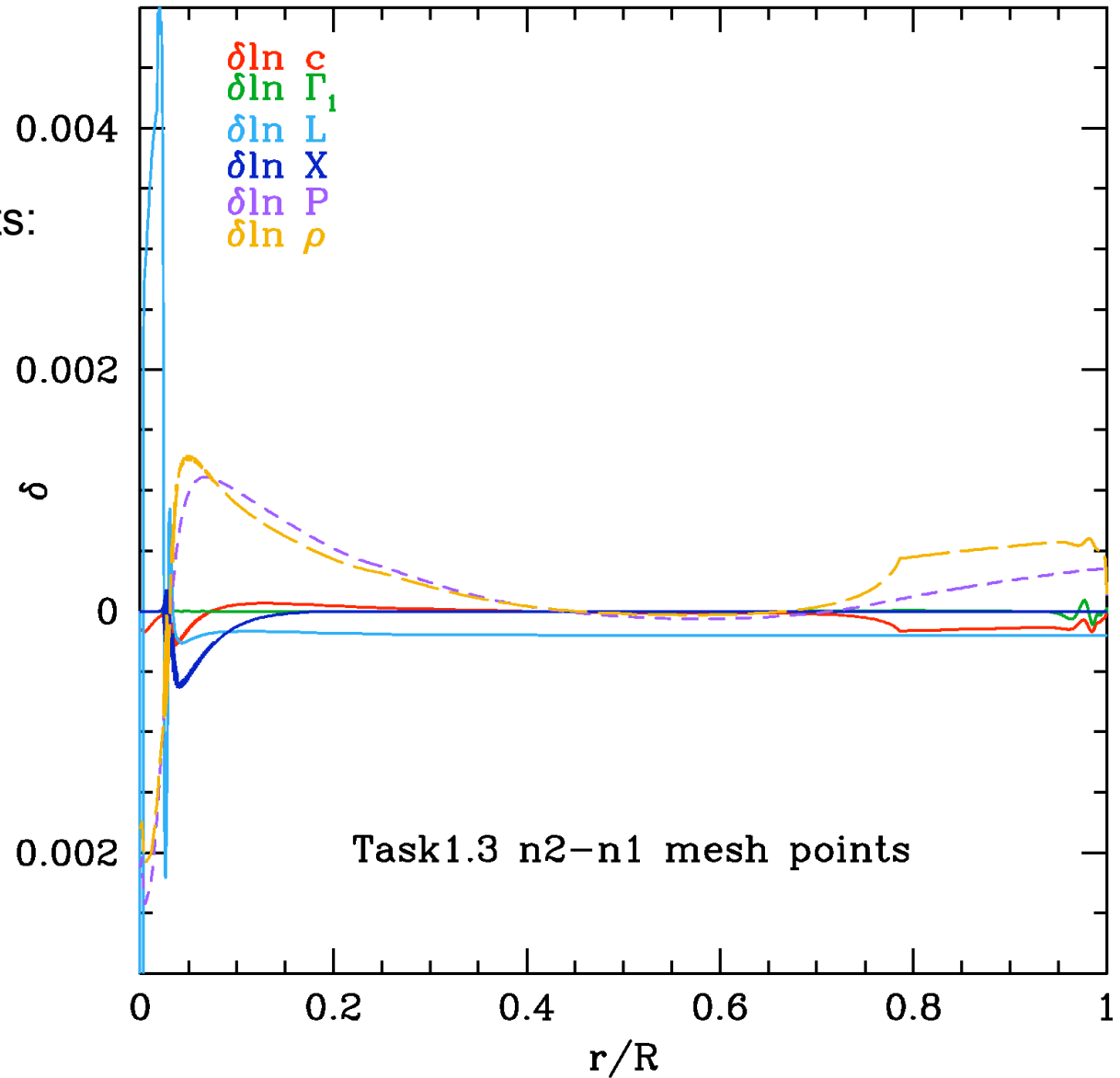


# Case 1.3

## CLES

1.2  $M_{-}$ ,  $M_c = 0.1 M_{-}$

Test effect of no. of meshpoints:  
(  $N = 2361$  ) – (  $N = 1187$  )

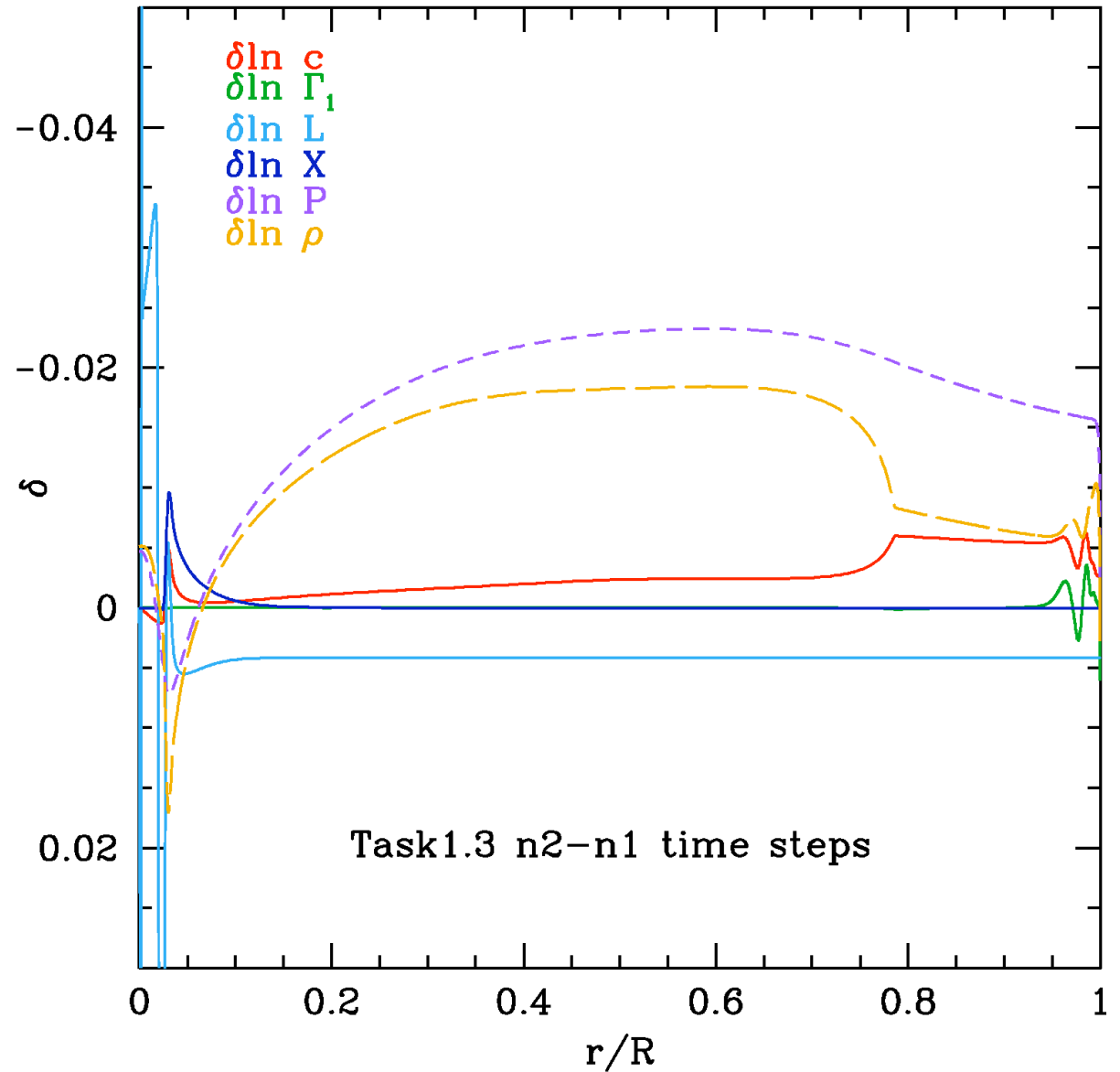


# Case 1.3

$1.2 M_{\odot}$ ,  $M_c = 0.1 M_{\odot}$

## CLES

Test effect of no. of timesteps:  
(  $N = 233$  ) – (  $N = 115$  )





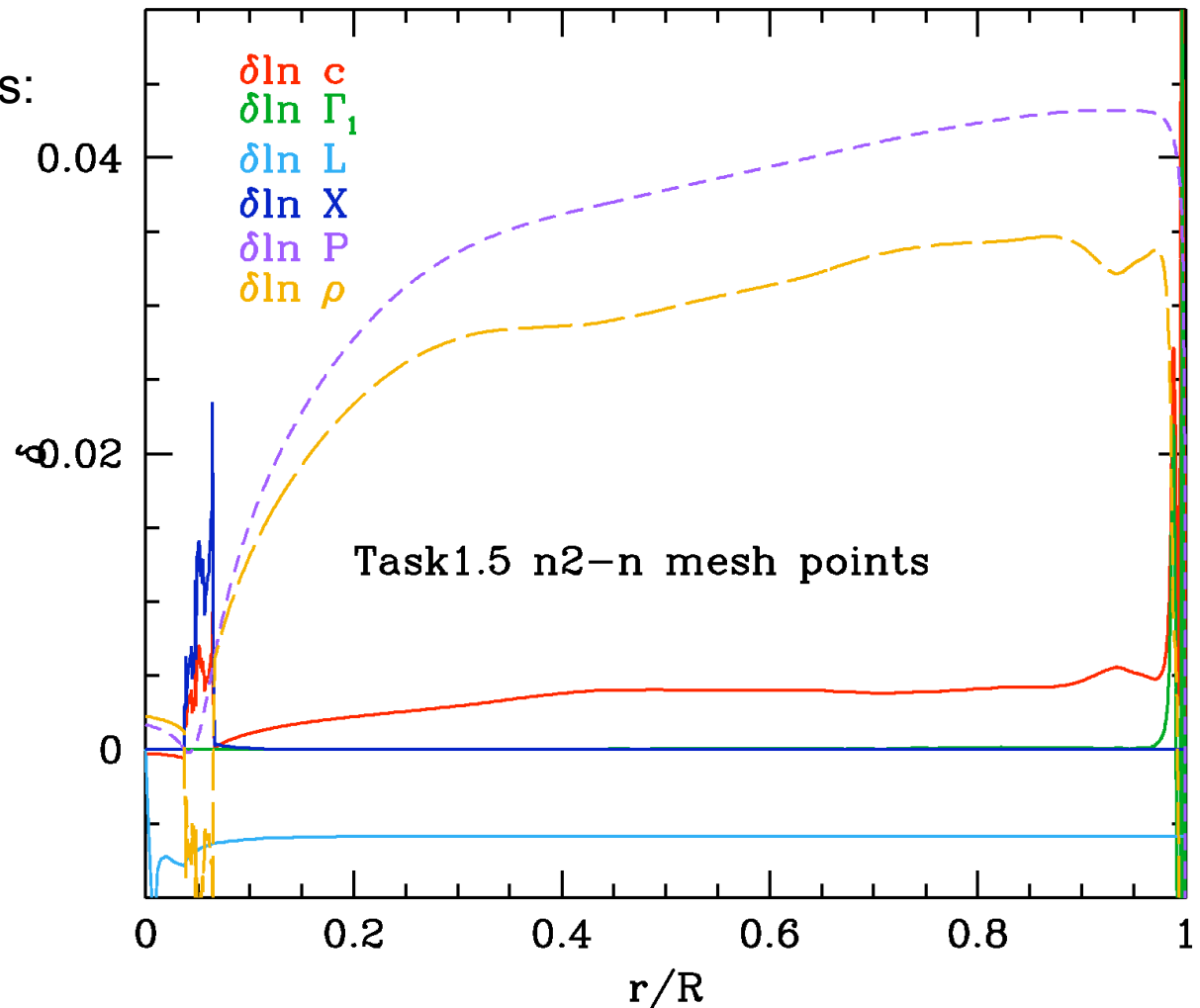
Case 1.5

$2.0 M_{\odot}$ ,  $X_c = 0.01$ ,

Overshoot  $0.15 H_p$

**CLES**

Test effect of no. of meshpoints:  
(  $N = 2409$  ) – (  $N = 1200$  )



Case 1.5

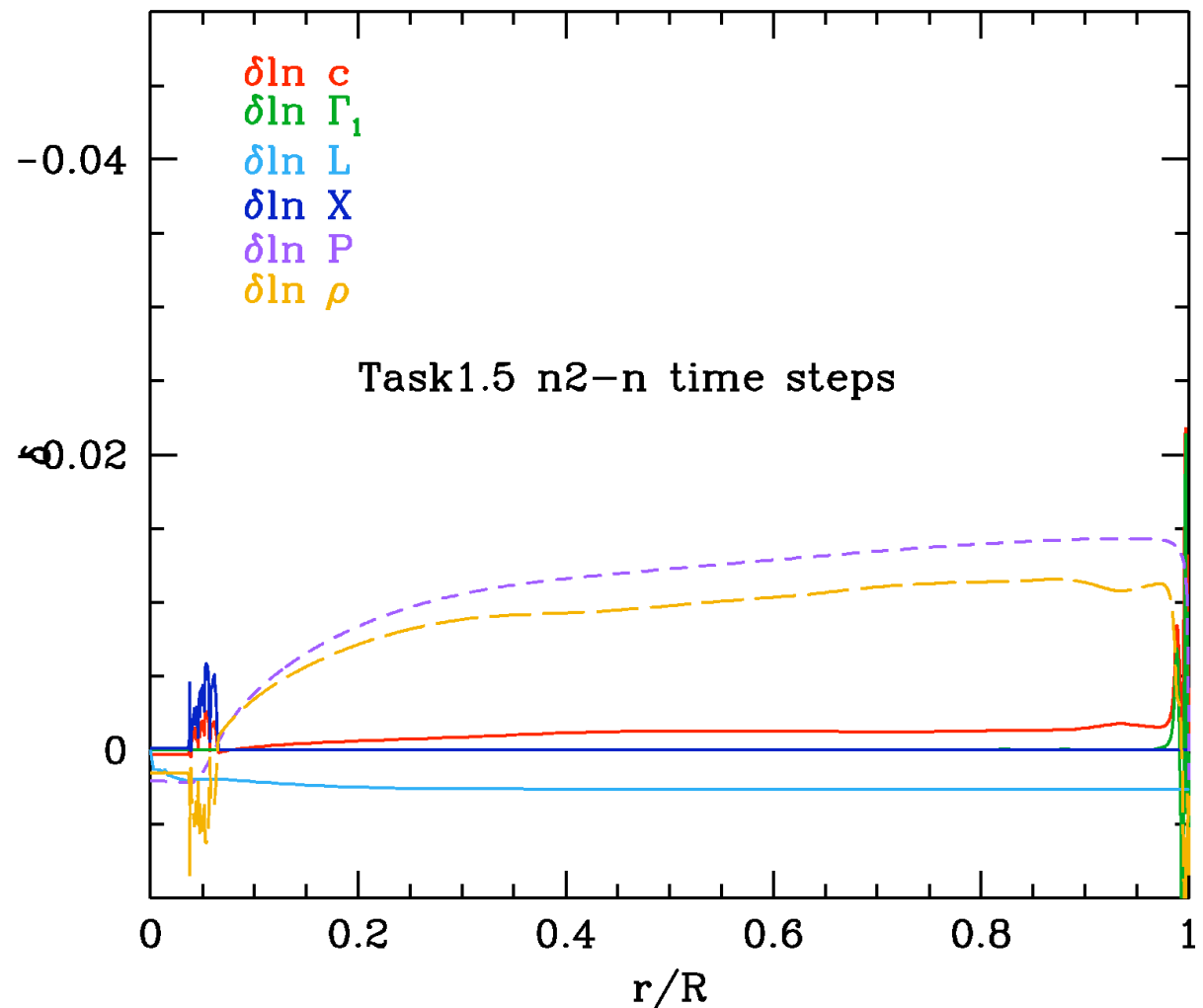
$2.0 M_{\odot}$ ,  $X_c = 0.01$ ,

Overshoot  $0.15 H_p$

**CLES**

Test effect of no. of timesteps:

( $N = 374$ ) – ( $N = 189$ )



# Physics comparisons

Evaluate physics (EOS, opacity, energy-generation rate, rate of composition change, ..., at fixed  $T$ ,  $\rho$ ,  $X_i$ )

Examples: comparing CESAM and CLES with ASTEC, showing, e.g.,

$$\ln(\kappa_{\text{ASTEC}}(\rho_{\text{CESAM}}, T_{\text{CESAM}}, \dots) / \kappa_{\text{CESAM}})$$

# CESAM, Case 1.1

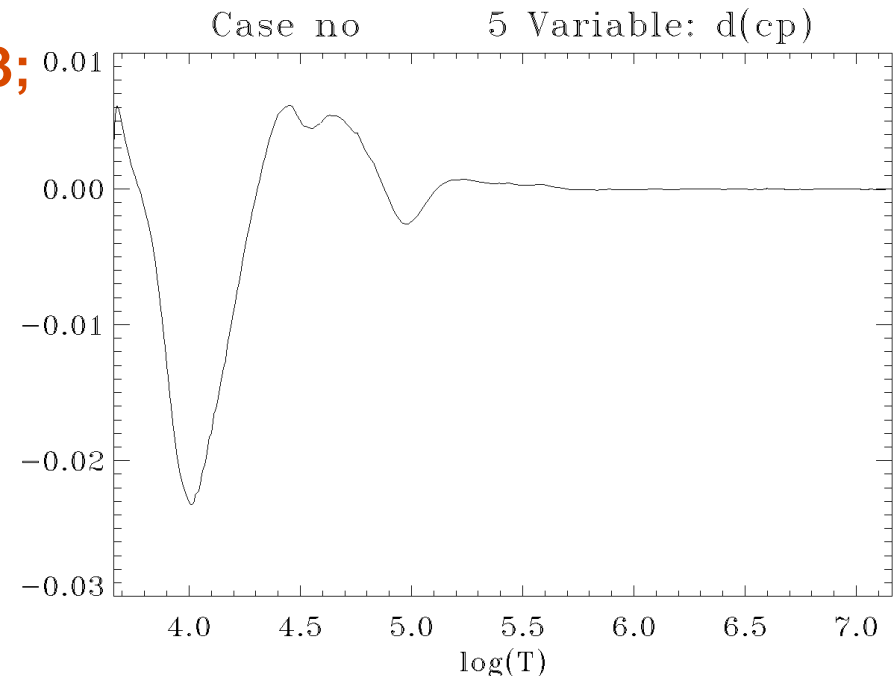
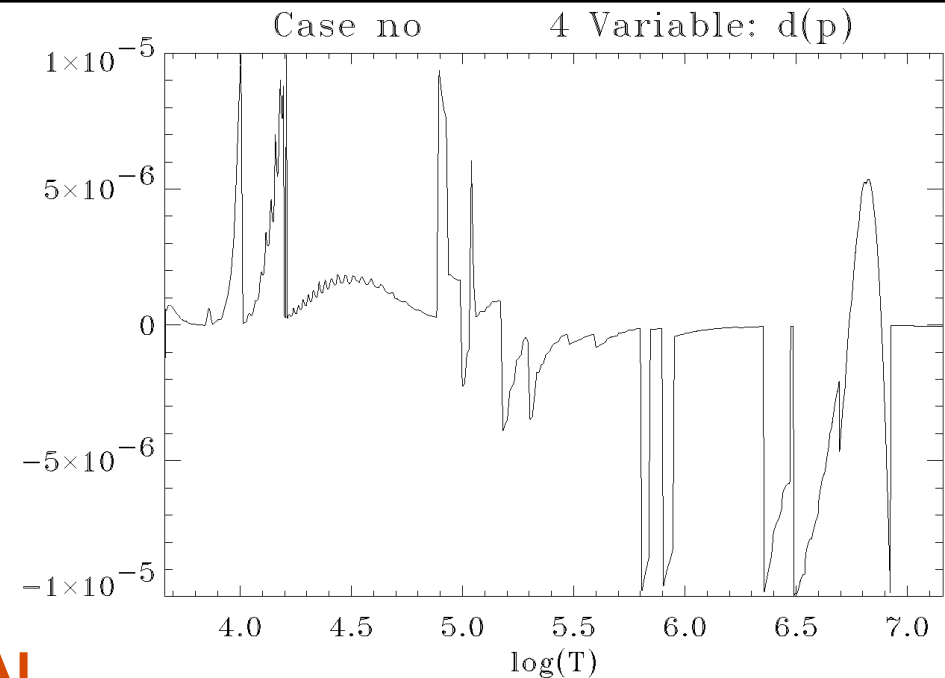
**Note: consistency problems in OPAL.**

**See also Boothroyd & Sackman (2003;  
ApJ 583, 1004)**

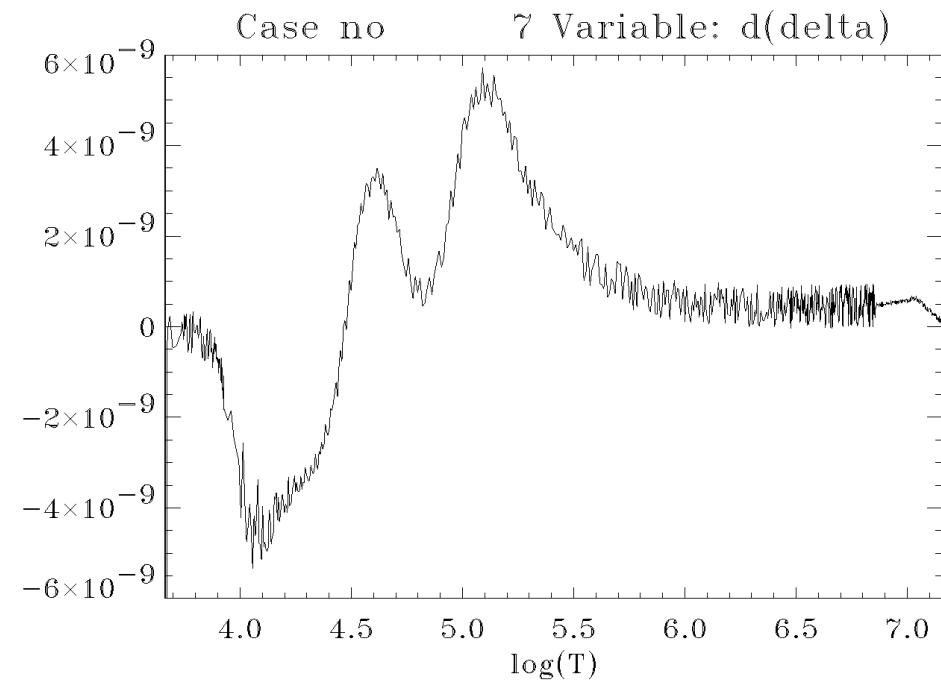
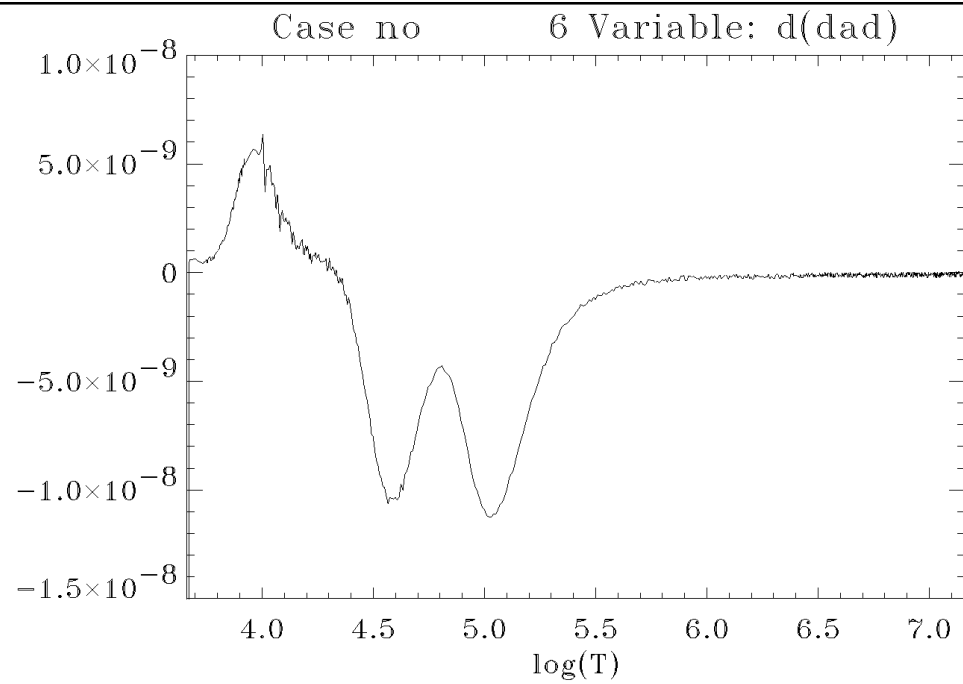
In ASTEC implementation:

Directly from OPAL:  $p$ ,  $r_{\text{ad}}$ ,  $\delta$ ,  $\alpha$ ,  $\Gamma_1$

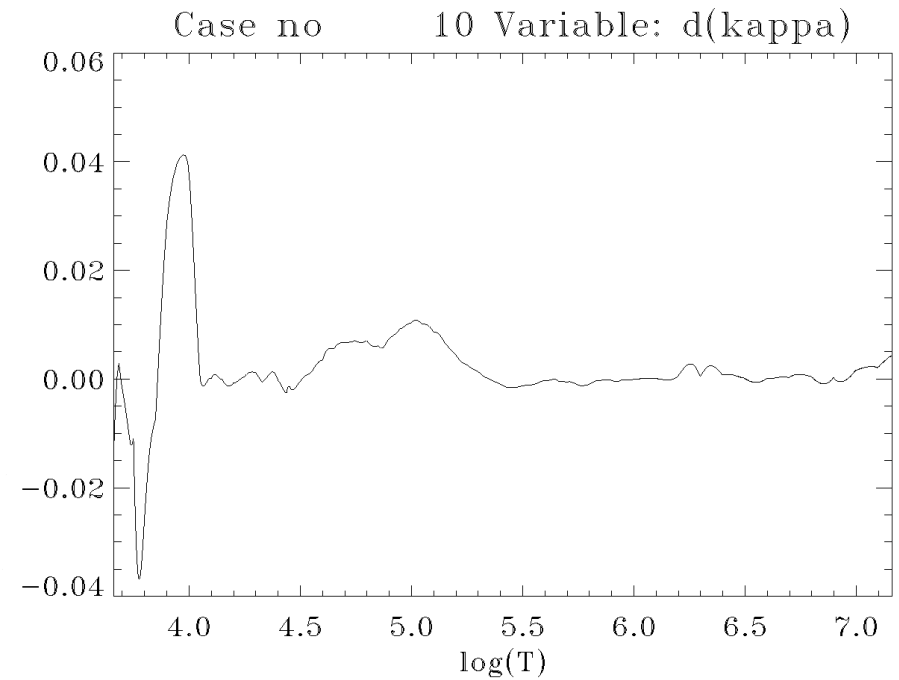
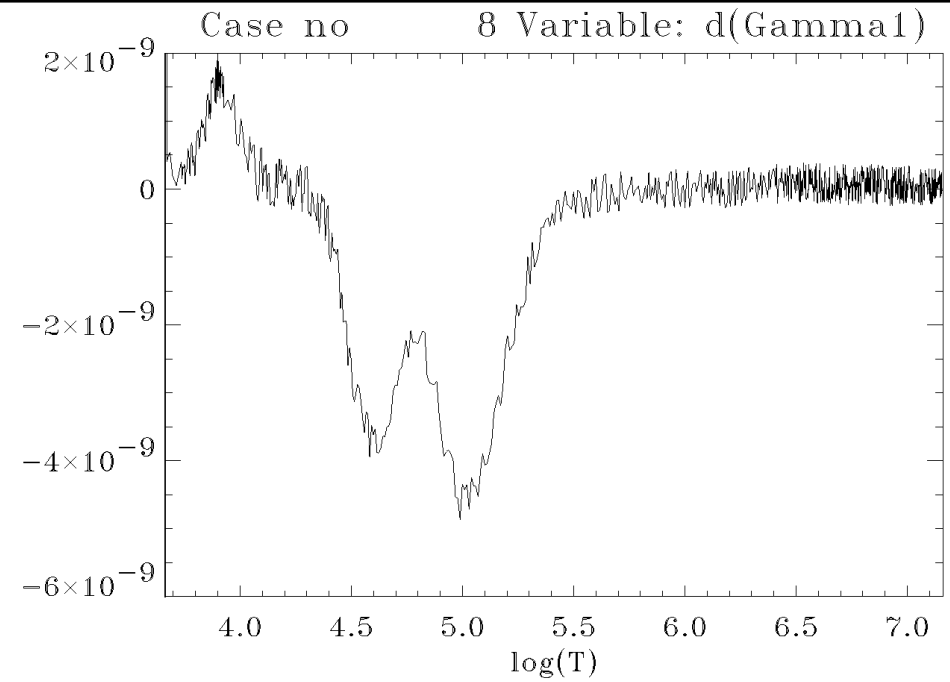
$$c_p = c_v + p \delta^2 / (\rho T \alpha)$$



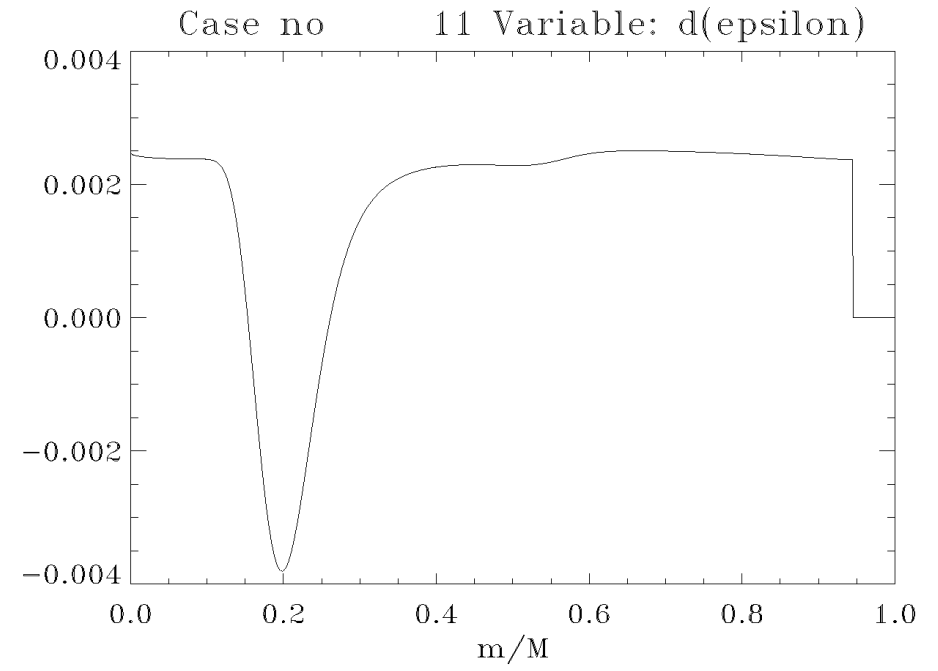
# CESAM, Case 1.1



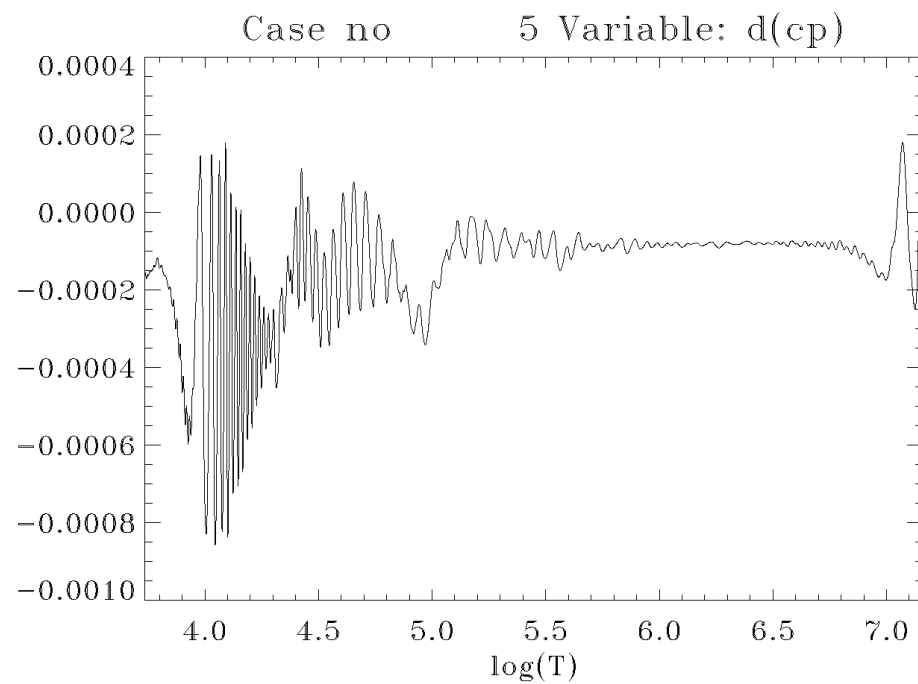
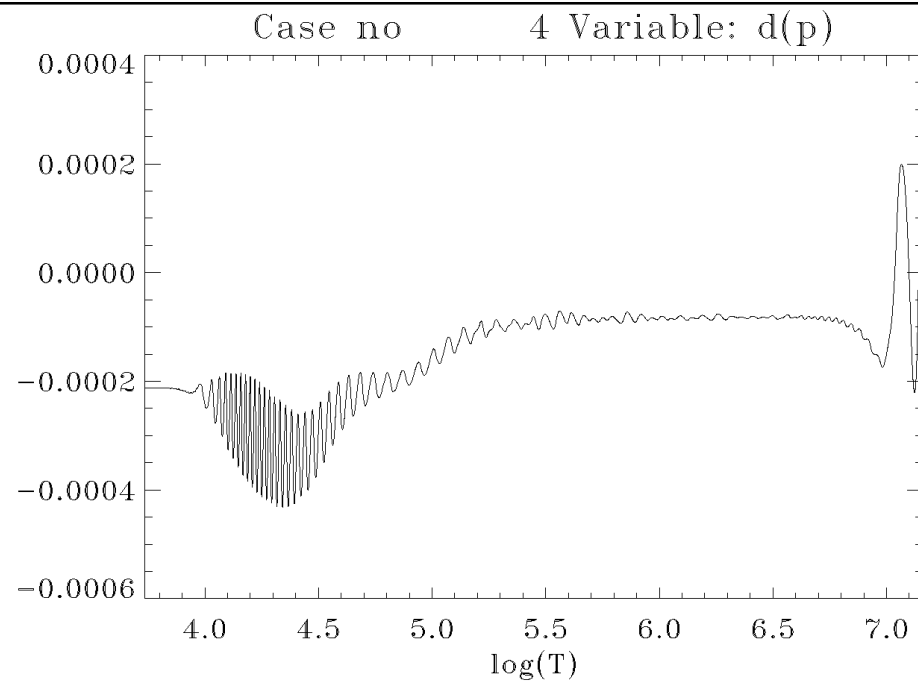
# CESAM, Case 1.1



# CESAM, Case 1.1

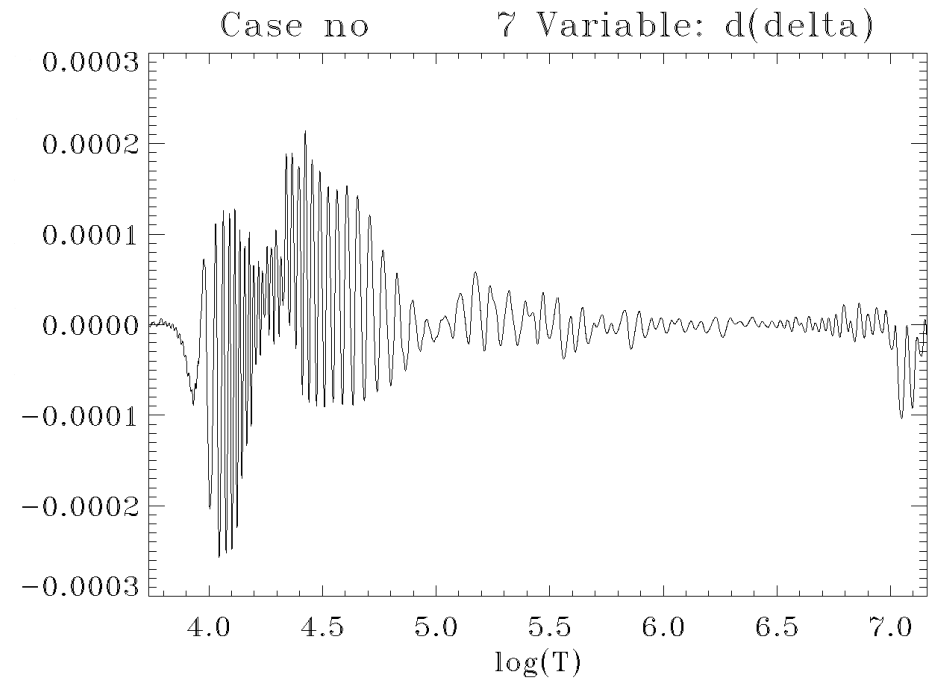
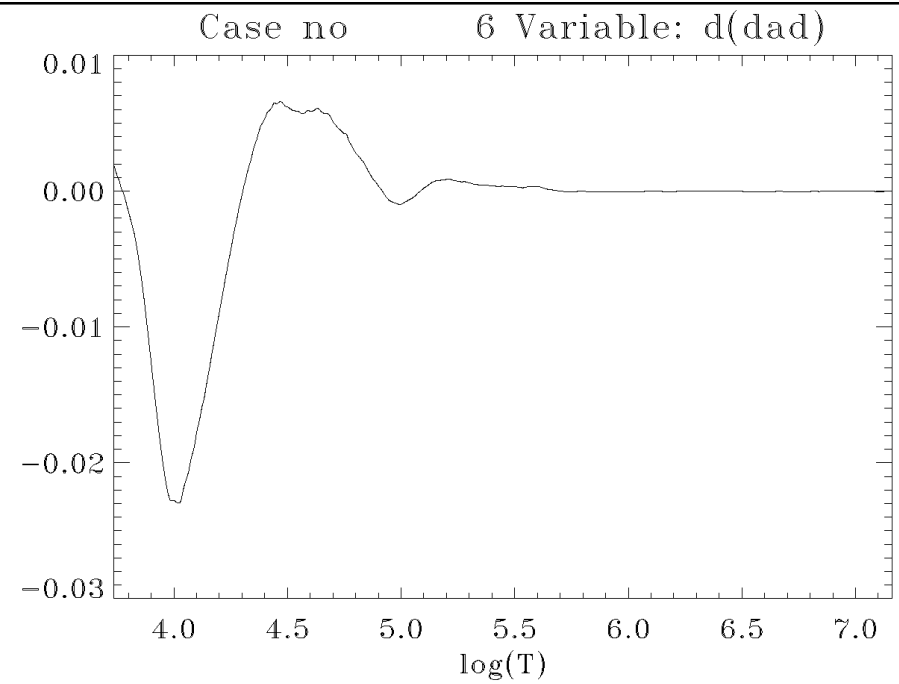


# CLES, Case 1.1



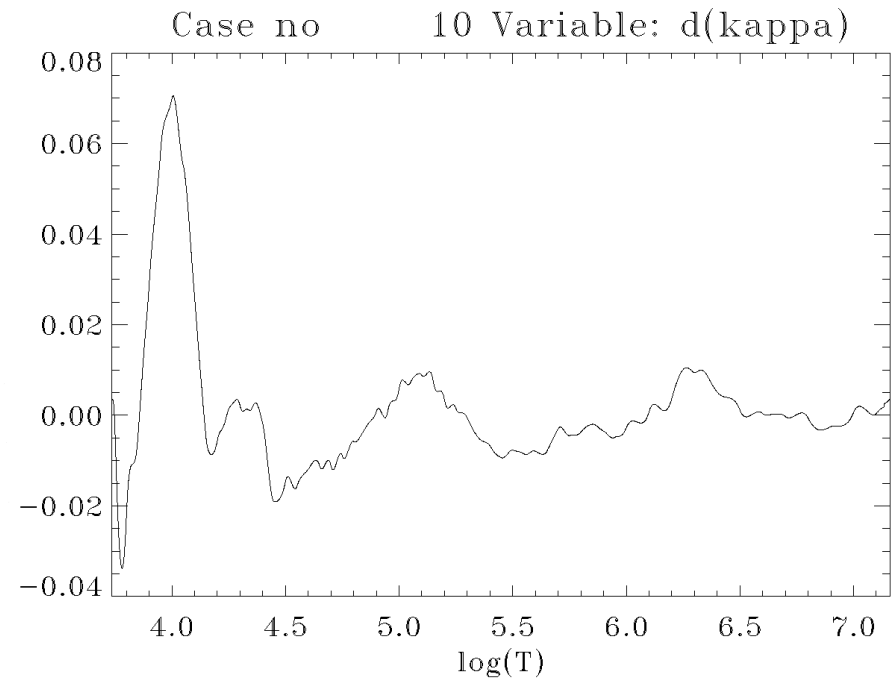
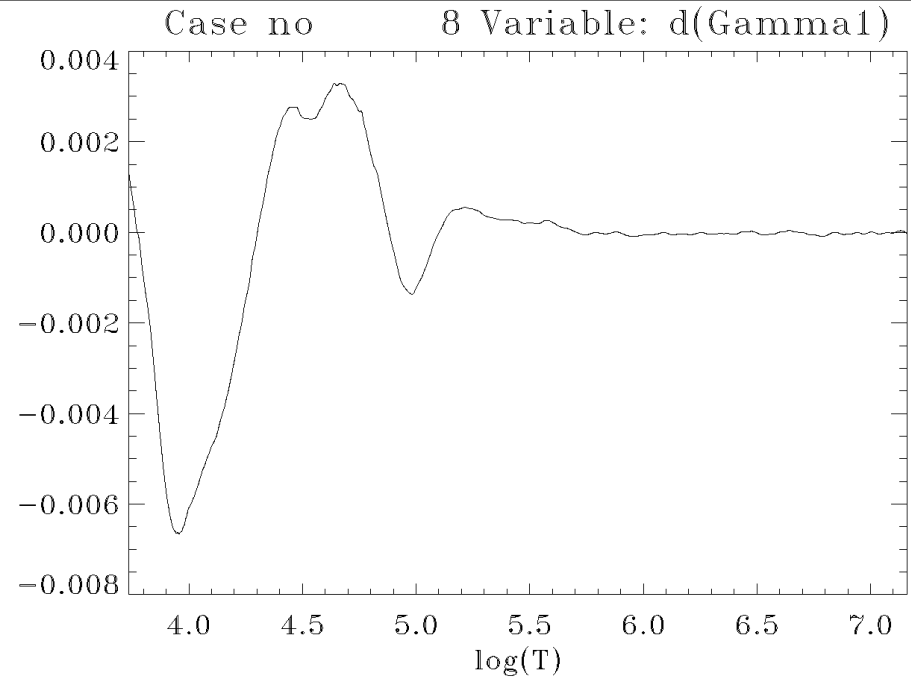


# CLES, Case 1.1



# CLES, Case 1.1

**OPAL 2005 appears to be much more consistent!**



# Effects of electron conduction

## Case 1.3

$M = 1.2 M_{\odot}, M_c = 0.1 M_{\odot}$

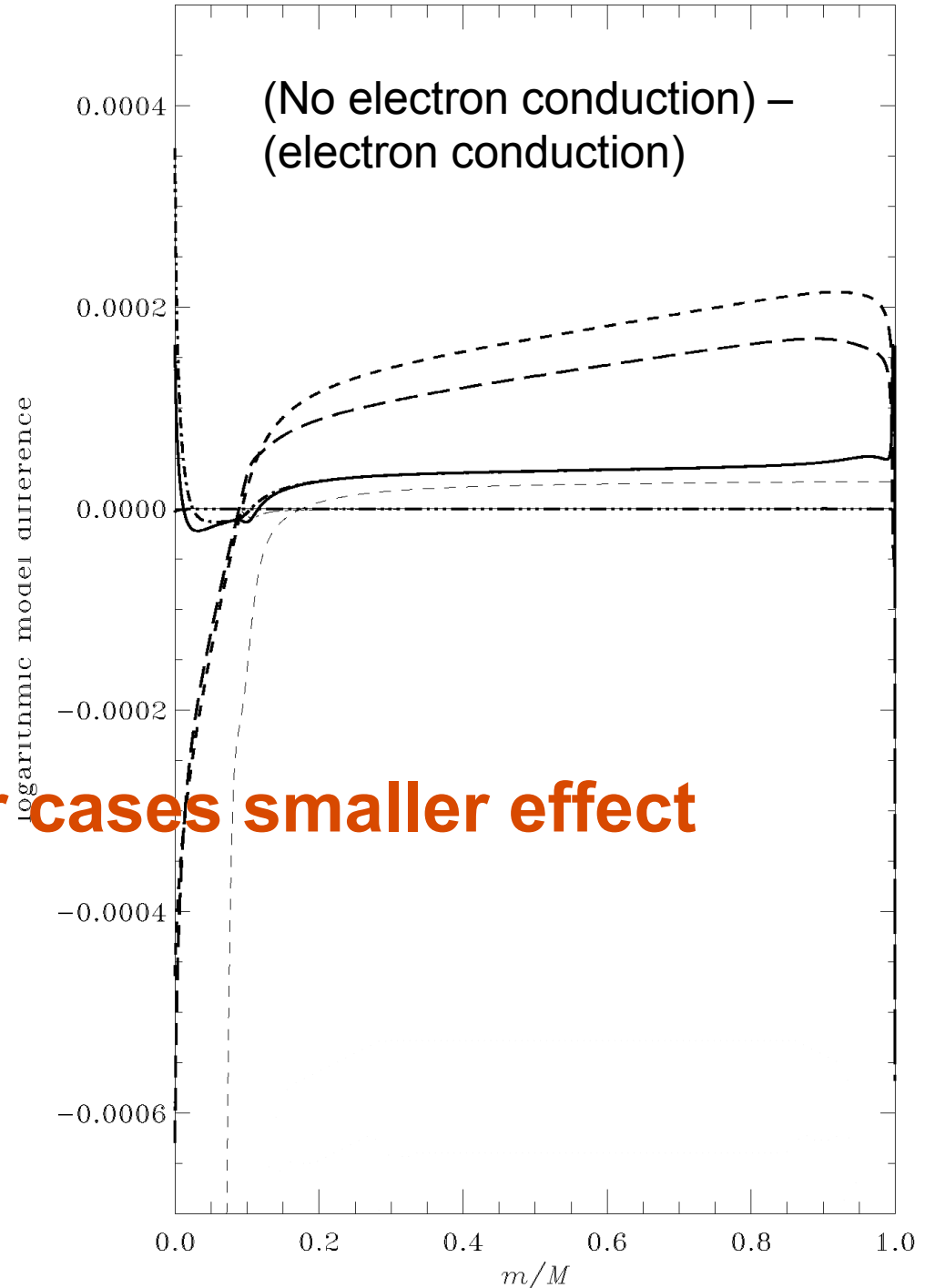
$Z = 0.01$

$\rho_c = 3253 \text{ g cm}^{-3}$

**In other cases smaller effect**

Line styles:

- |           |                         |             |                  |
|-----------|-------------------------|-------------|------------------|
| .....     | : $\delta \ln T$        | ————        | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | -----       | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | - · - · - · | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |             |                  |
| .....     | : $\delta \ln \Gamma_1$ |             |                  |



# Main project: compare different codes

- Evolution tracks
- Global parameters for selected models
- Detailed comparison of structure
- Comparison of oscillation frequencies

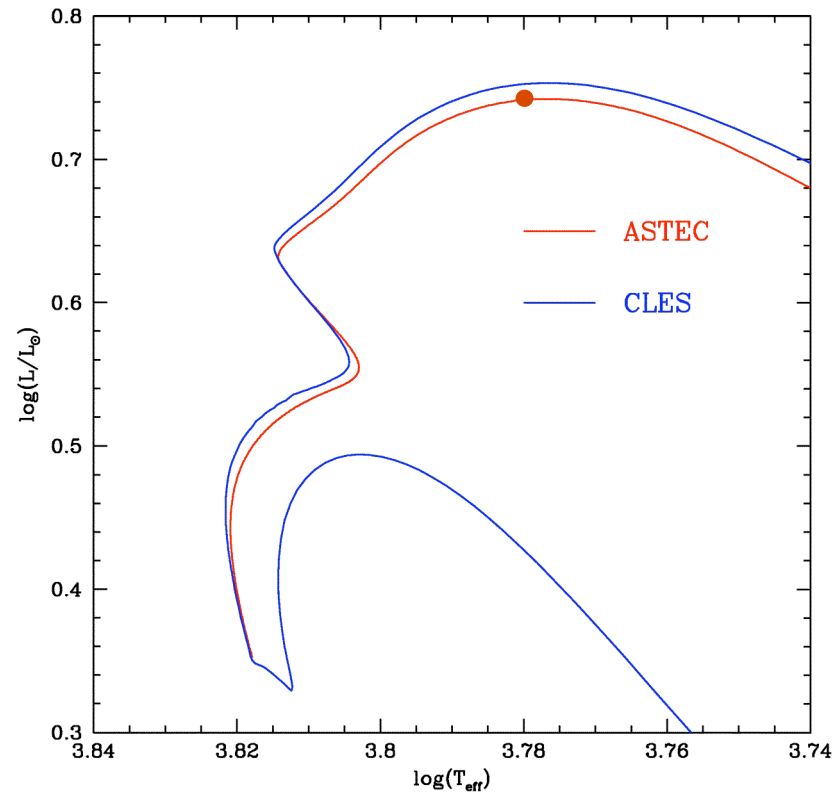
# CLES and ASTEC

Case 1.3

1.2 M<sub>⊙</sub>

$X_0 = 0.73$ ,  $Z_0 = 0.01$

$M_{\text{HeC}} = 0.1 M_{\text{⊙}}$



# CLES and ASTEC

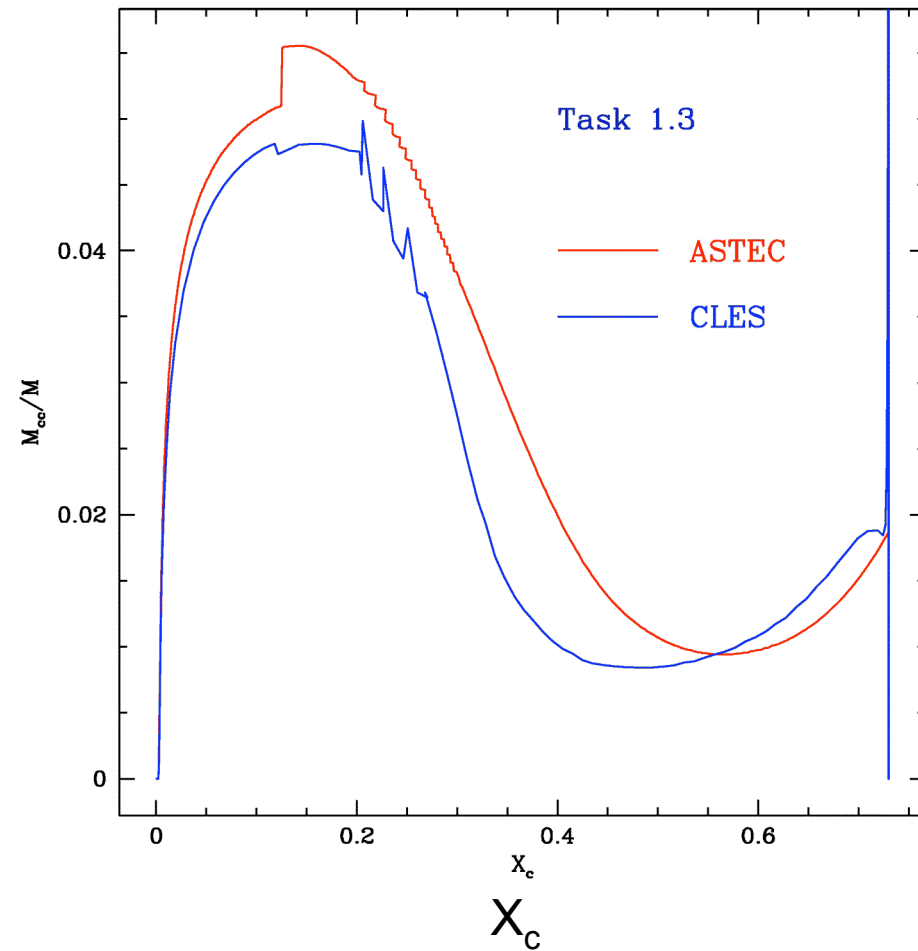
Case 1.3

1.2 M<sub>-</sub>

$X_0 = 0.73, Z_0 = 0.01$

$M_{\text{HeC}} = 0.1 M_{-}$

$M_c/M$



# CLES, CESAM and ASTEC

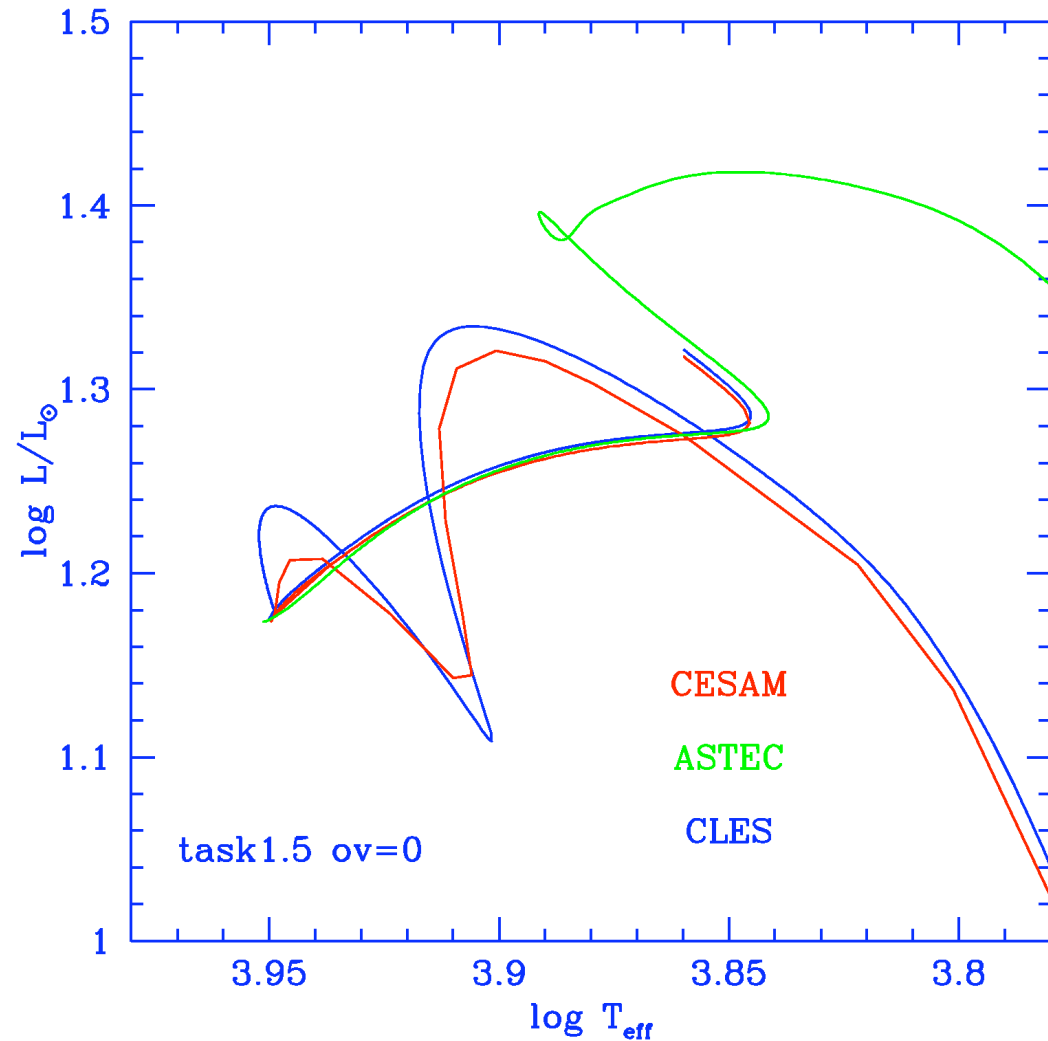
Case 1.5n

2.0 M-

$X_0 = 0.72$ ,  $Z_0 = 0.02$

$X_c = 0.01$

No overshoot



# CLES, CESAM and ASTEC

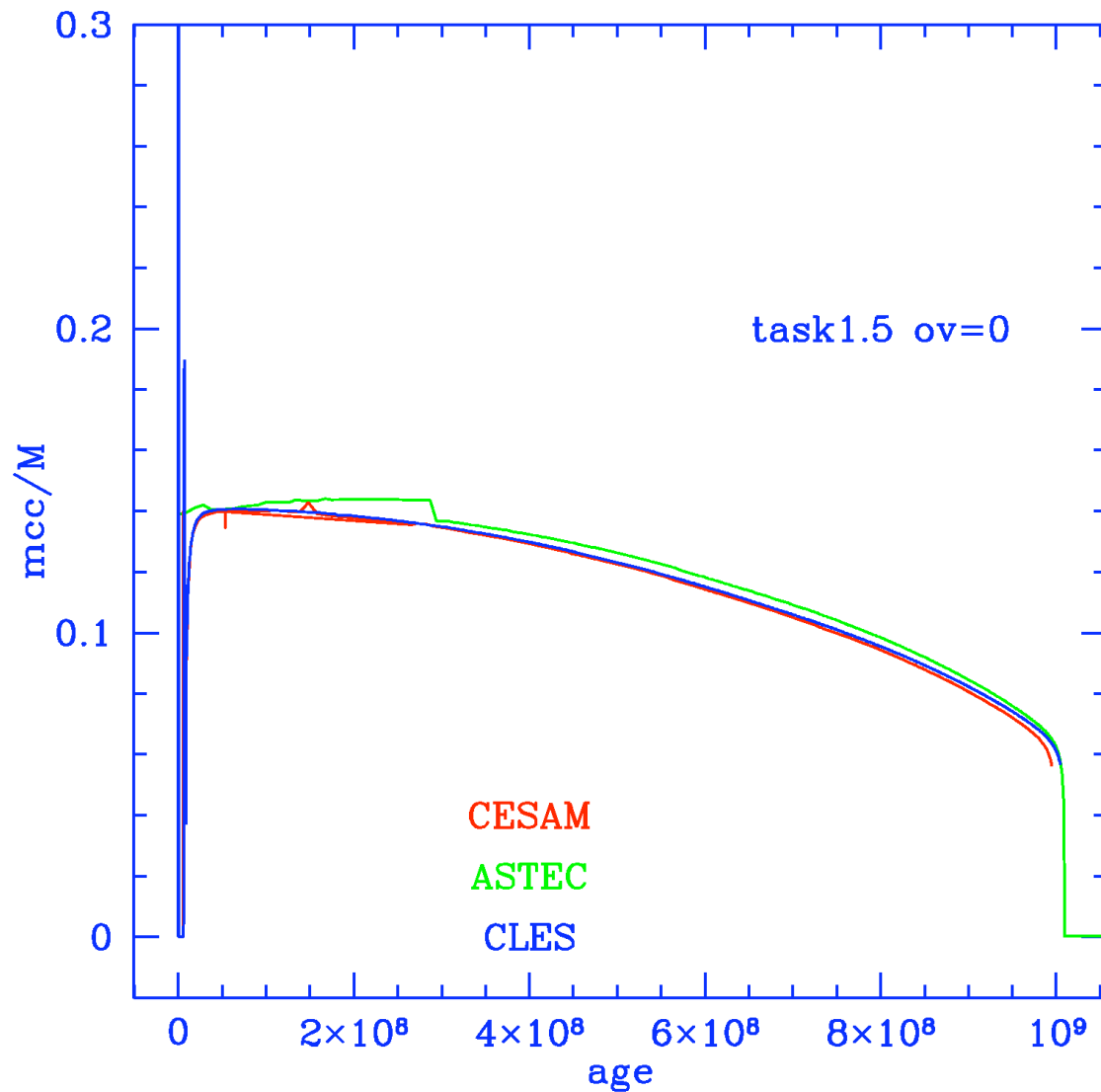
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# CLES, CESAM and ASTEC

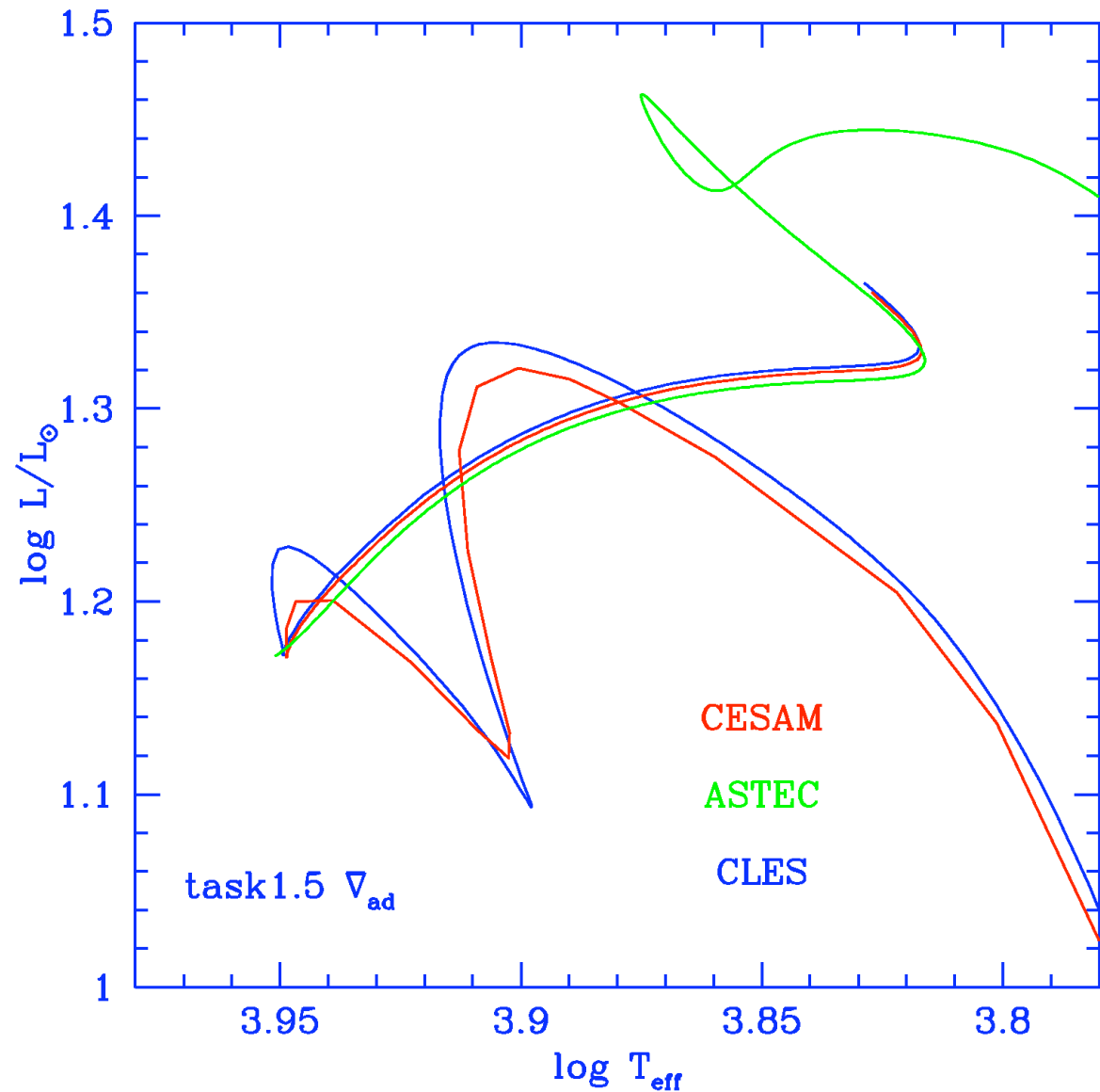
Case 1.5n

2.0 M-

$X_0 = 0.72$ ,  $Z_0 = 0.02$

$X_c = 0.01$

Overshoot  $0.15 H_p$



# CLES, CESAM and ASTEC

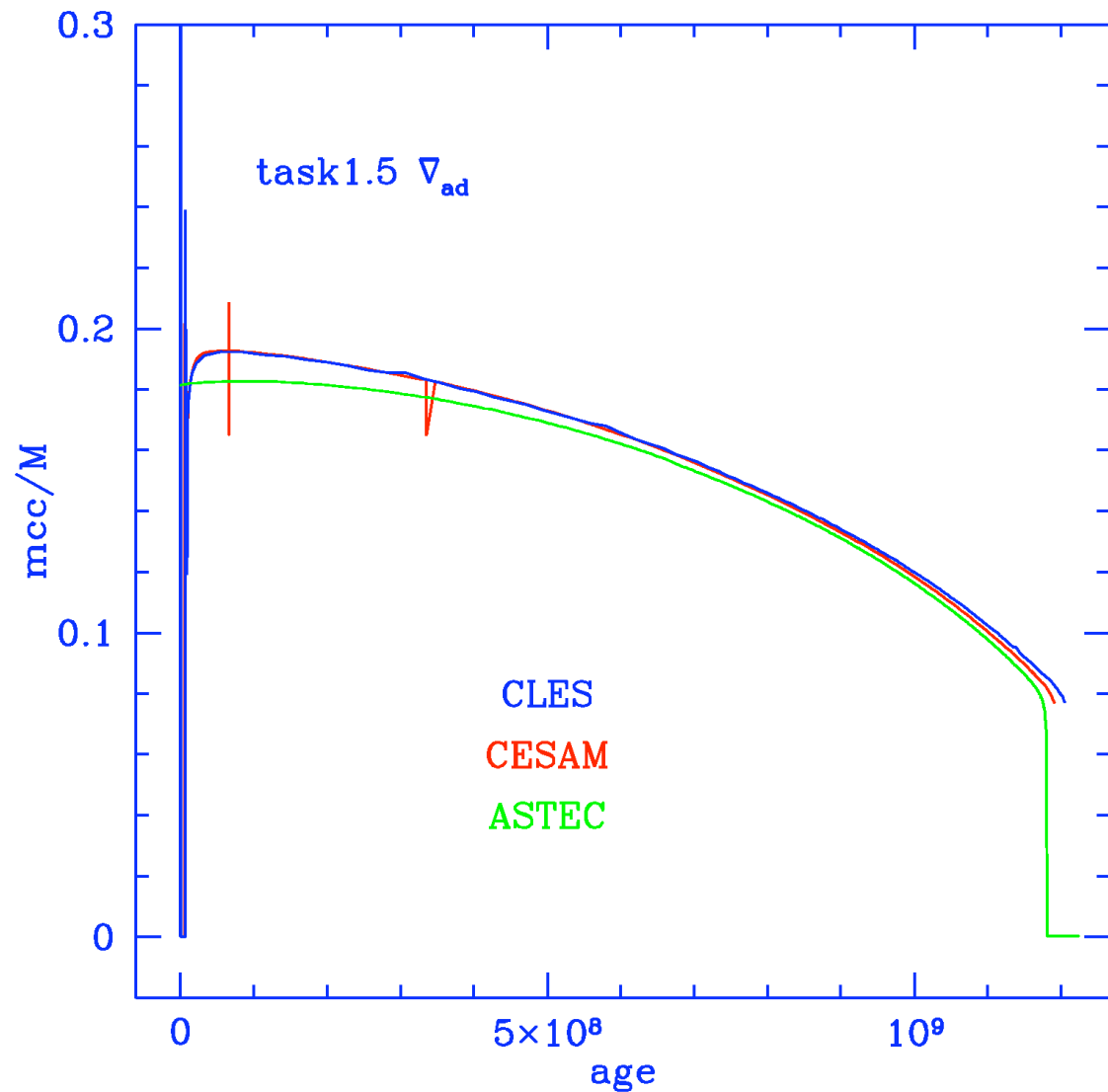
Case 1.5n

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$X_c = 0.01$

Overshoot  $0.15 H_p$



# Detailed model comparison

- Global quantities
- Differences at fixed  $m/M$ , plotted against  $m/M$  or  $r/R$
- Differences at fixed  $r/R$  might be more illustrative for effects on oscillations (but not used yet)

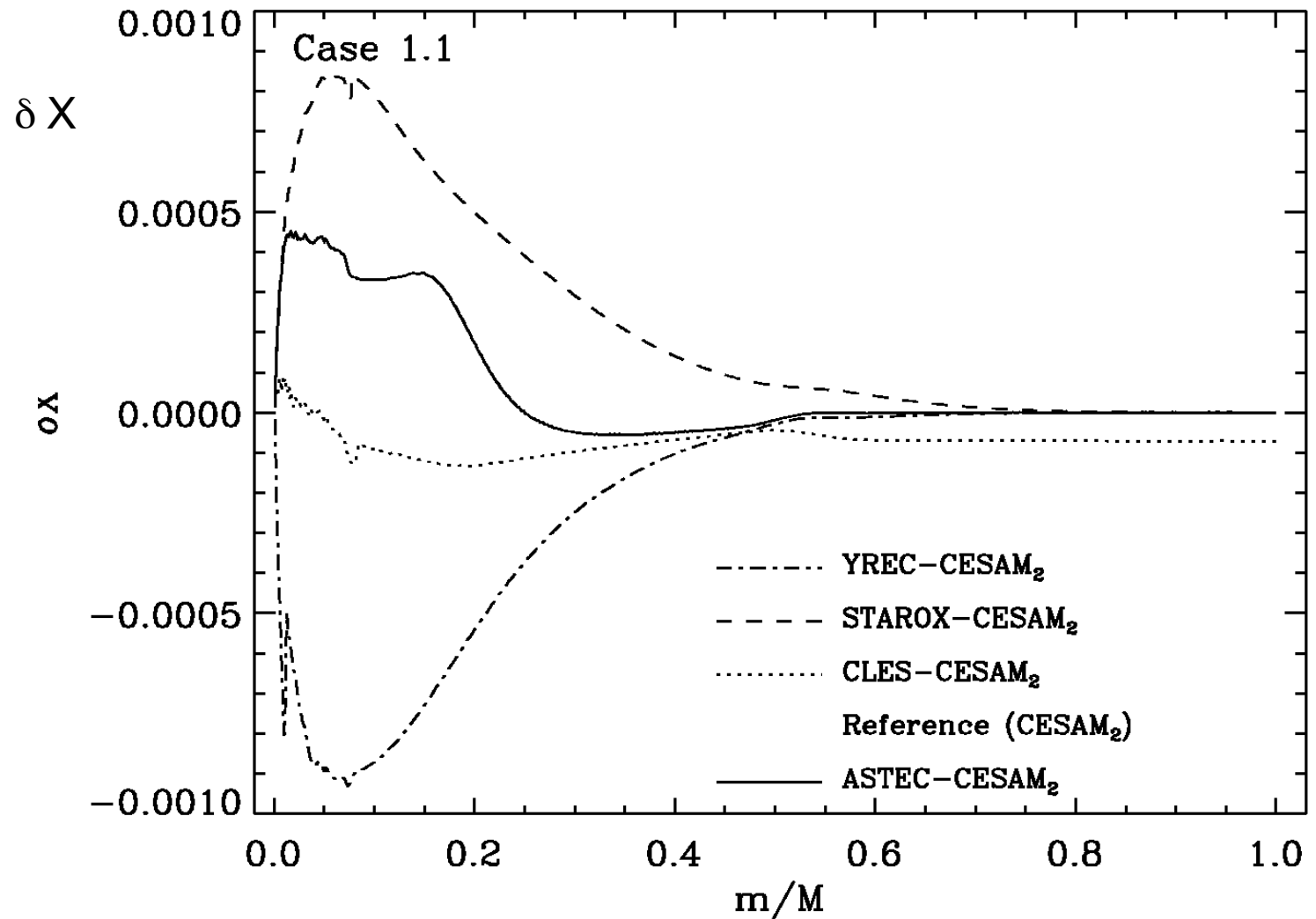
# Hydrogen abundance

0.9 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.35$



# Case 1.1

$0.9 M_{\odot}$ ,  $X_c = 0.35$

Line styles:

..... :  $\delta \ln T$

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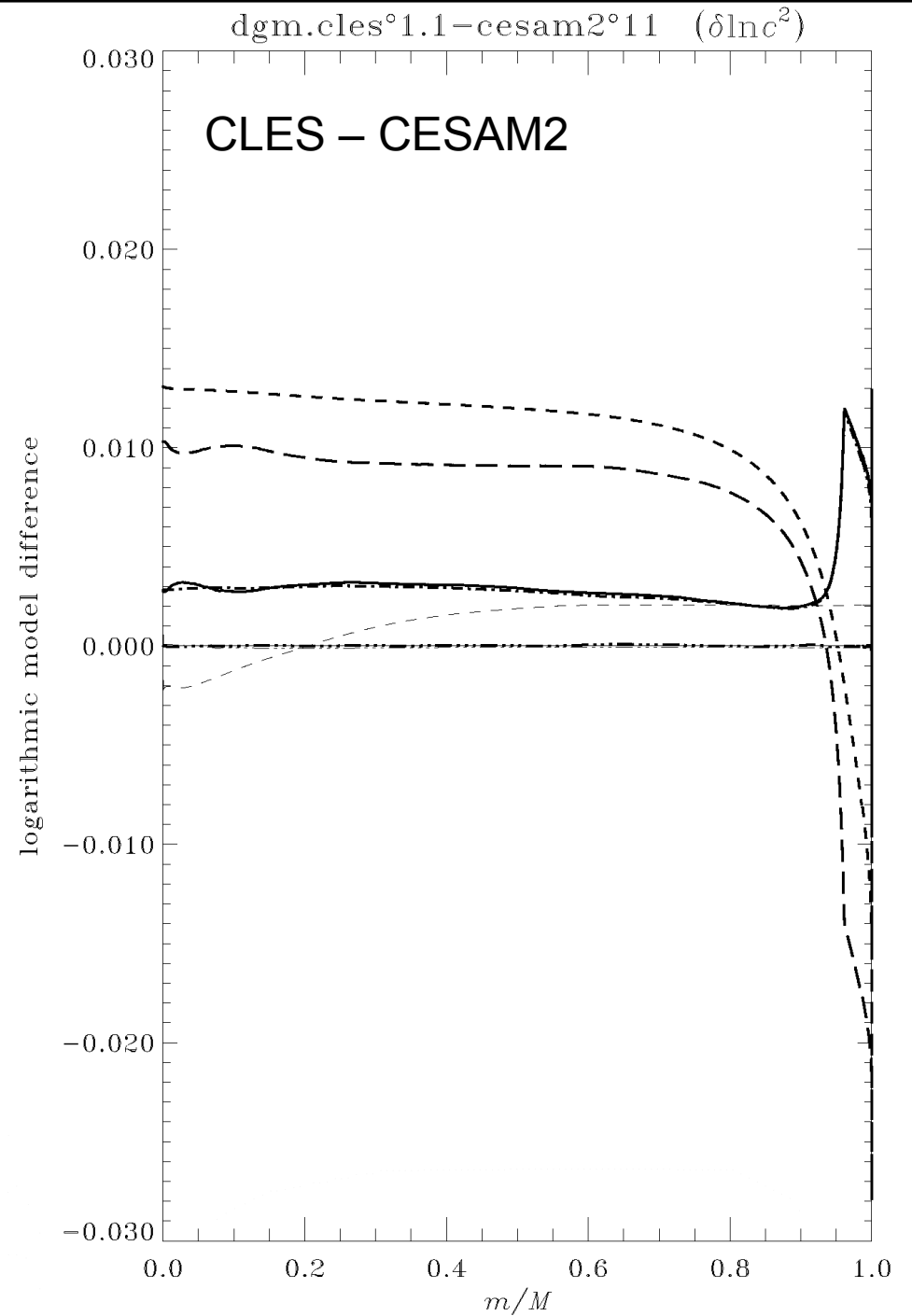
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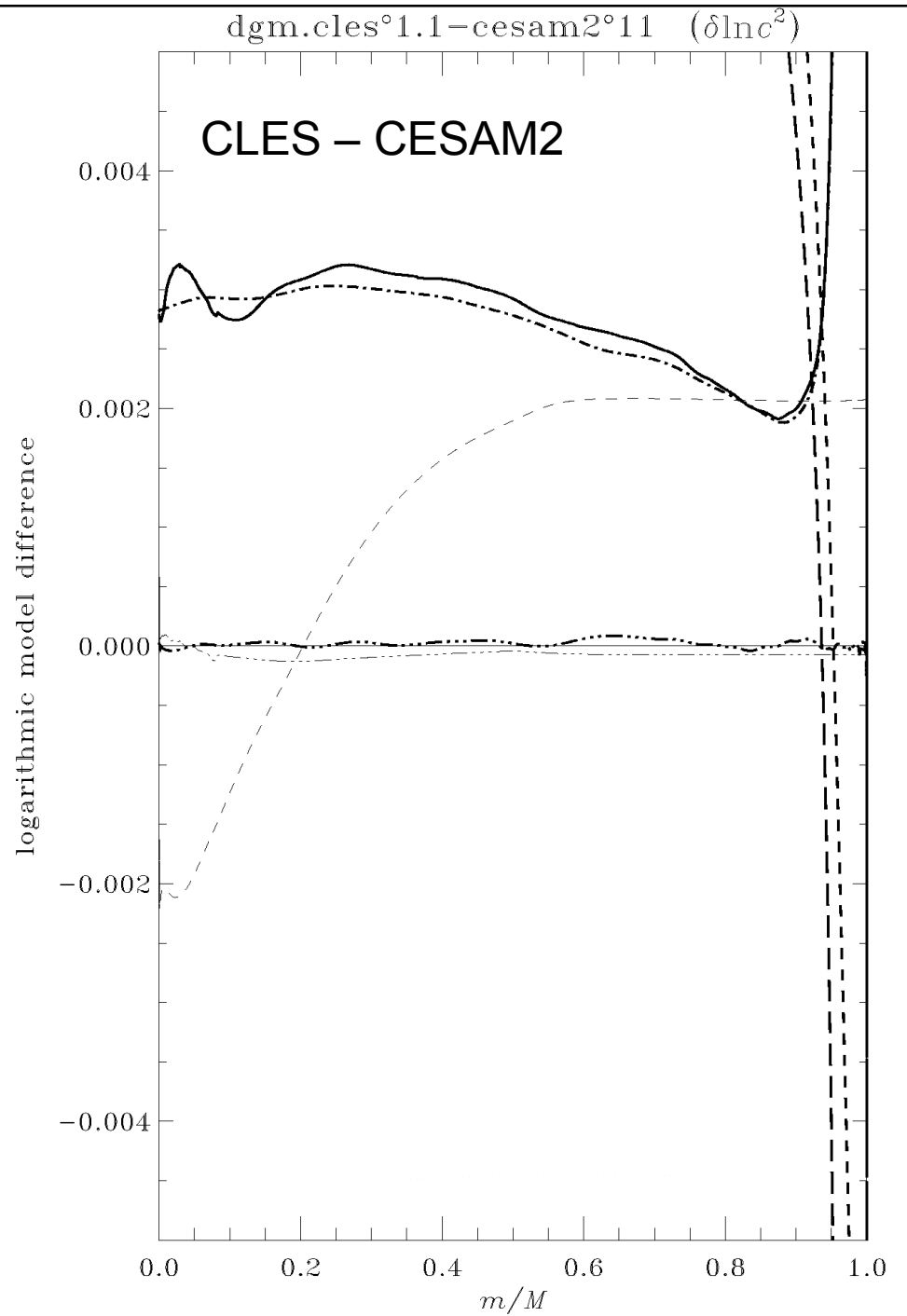
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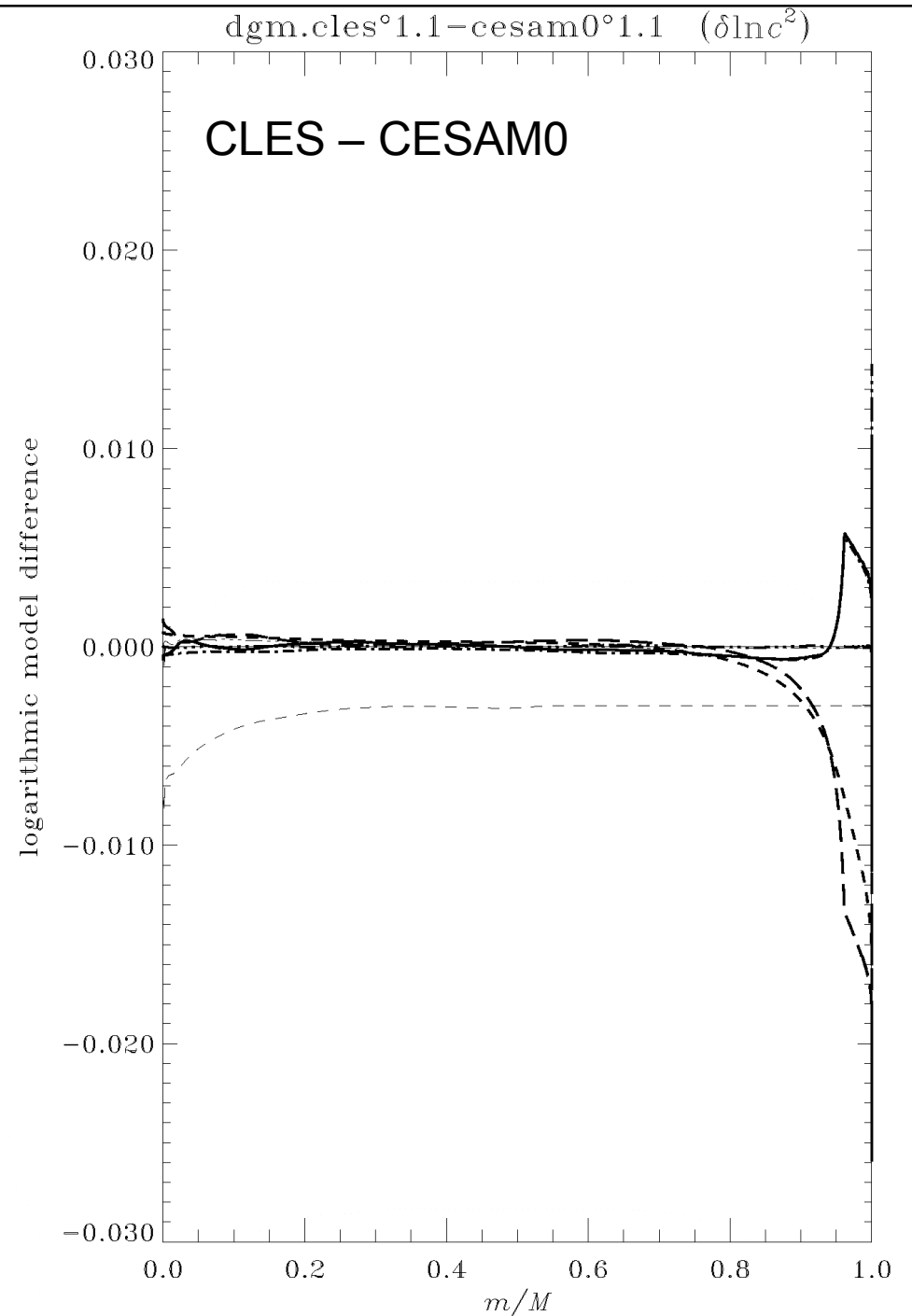
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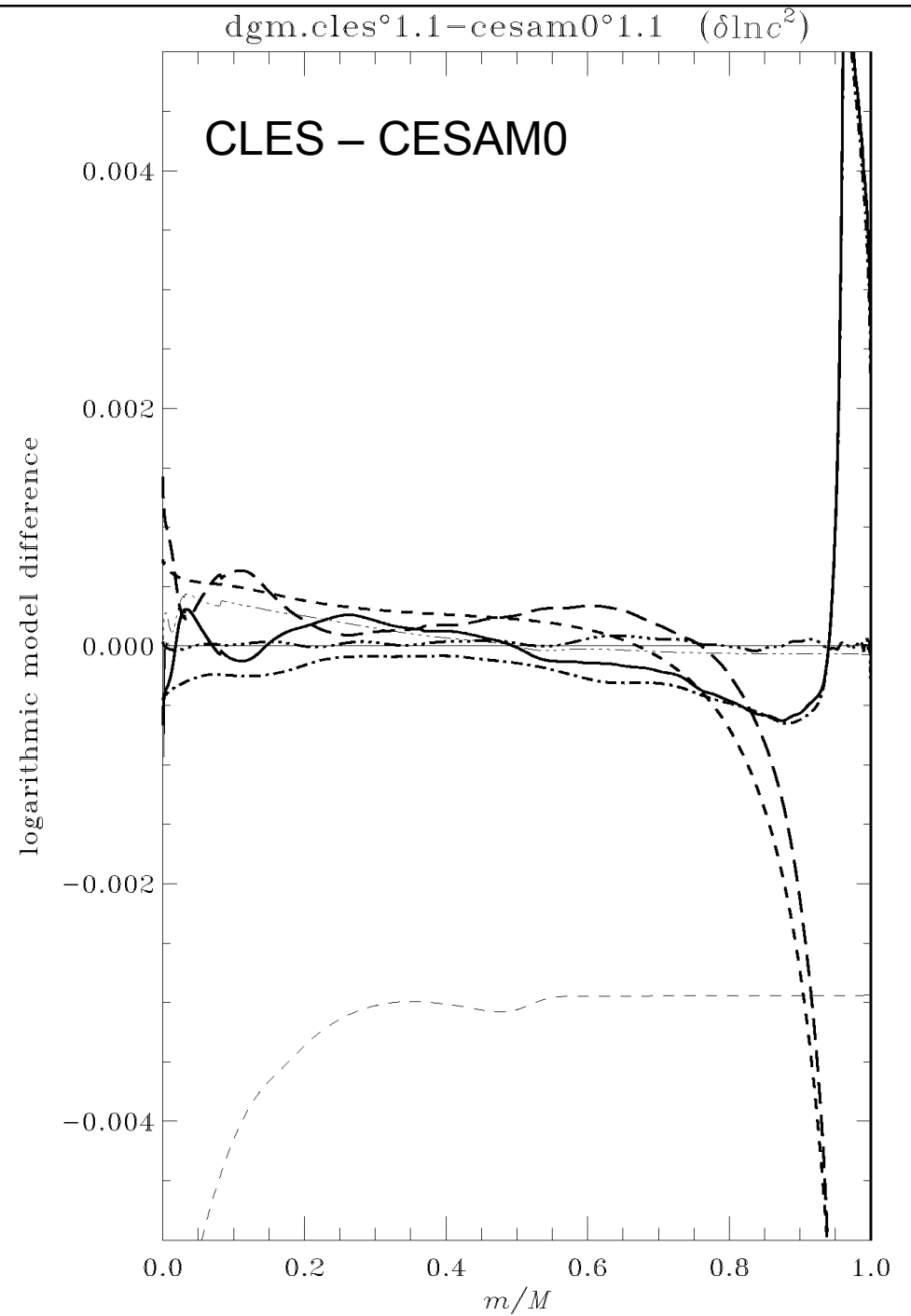
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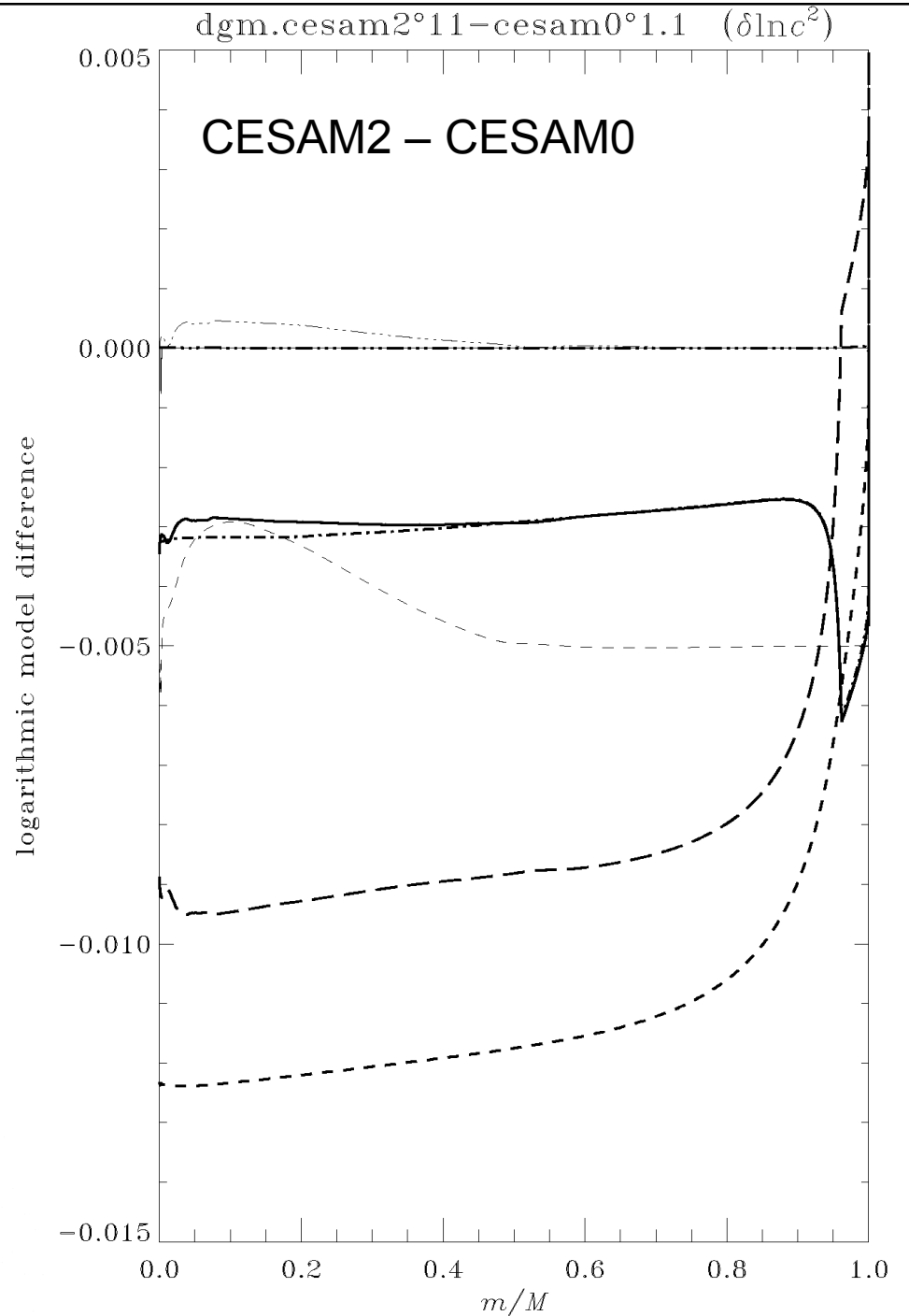


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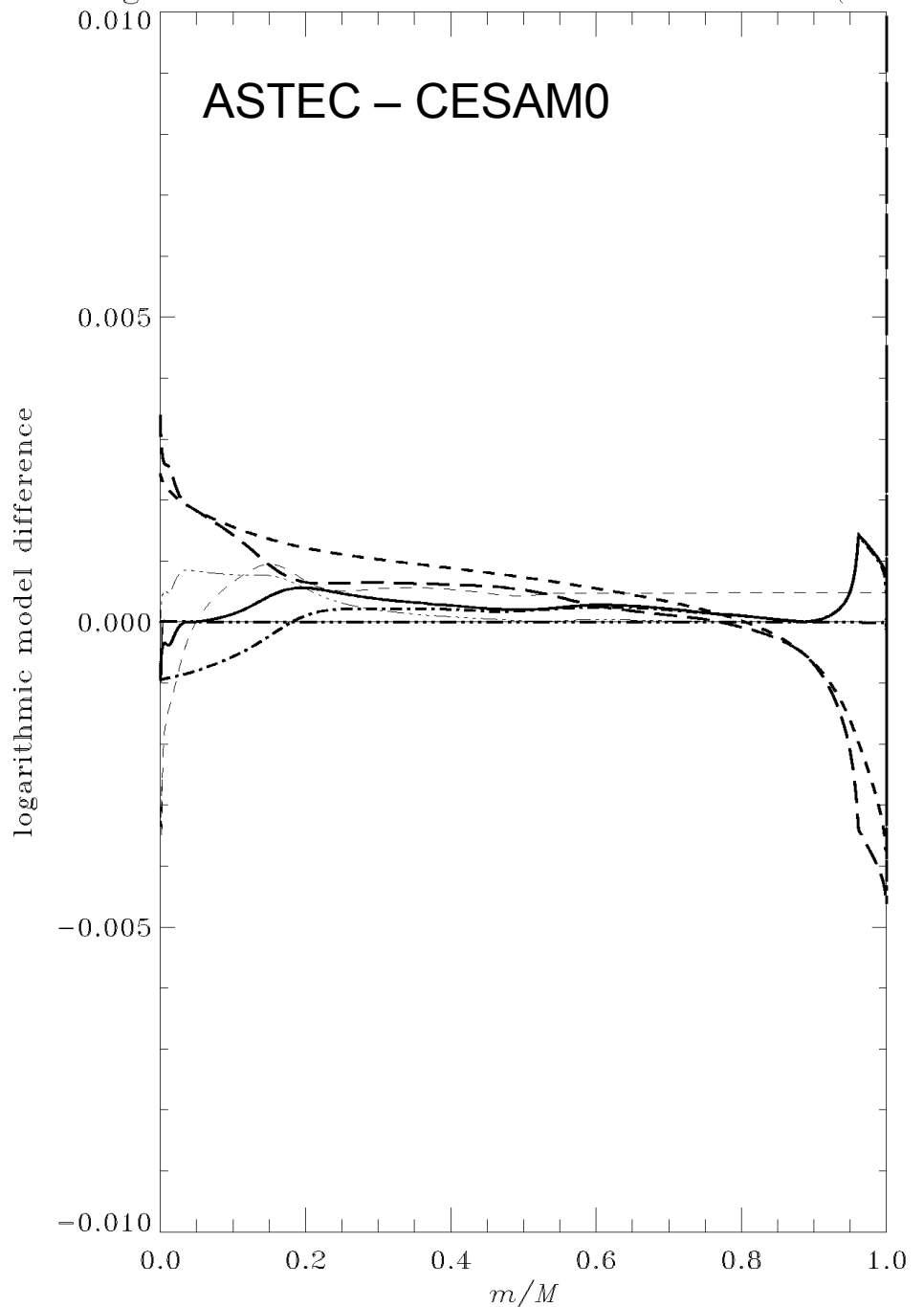
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Line styles:

- |           |                         |           |                  |
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| ————      | : $\delta \ln c^2$      |           |                  |
| .....     | : $\delta \ln \Gamma_1$ |           |                  |

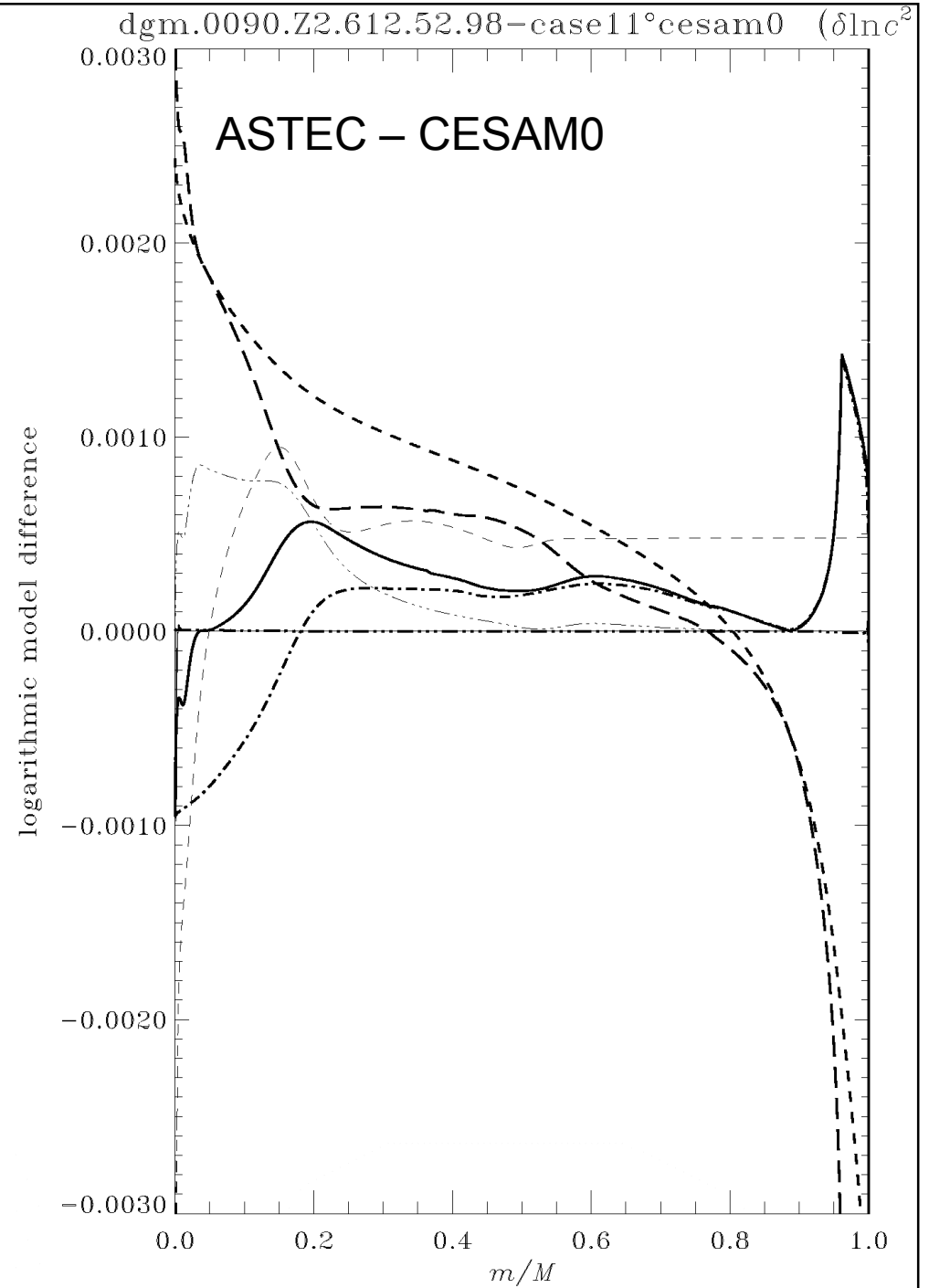
# Case 1.1

$0.9 M_{\odot}$ ,  $X_c = 0.35$

Line styles:

..... :  $\delta \ln T$   
----- :  $\delta \ln p$   
- - - - :  $\delta \ln \rho$   
———— :  $\delta \ln c^2$   
- · - · - :  $\delta \ln \Gamma_1$

———— :  $\delta \ln q$   
----- :  $\delta \ln L$   
- · - · - :  $\delta X$



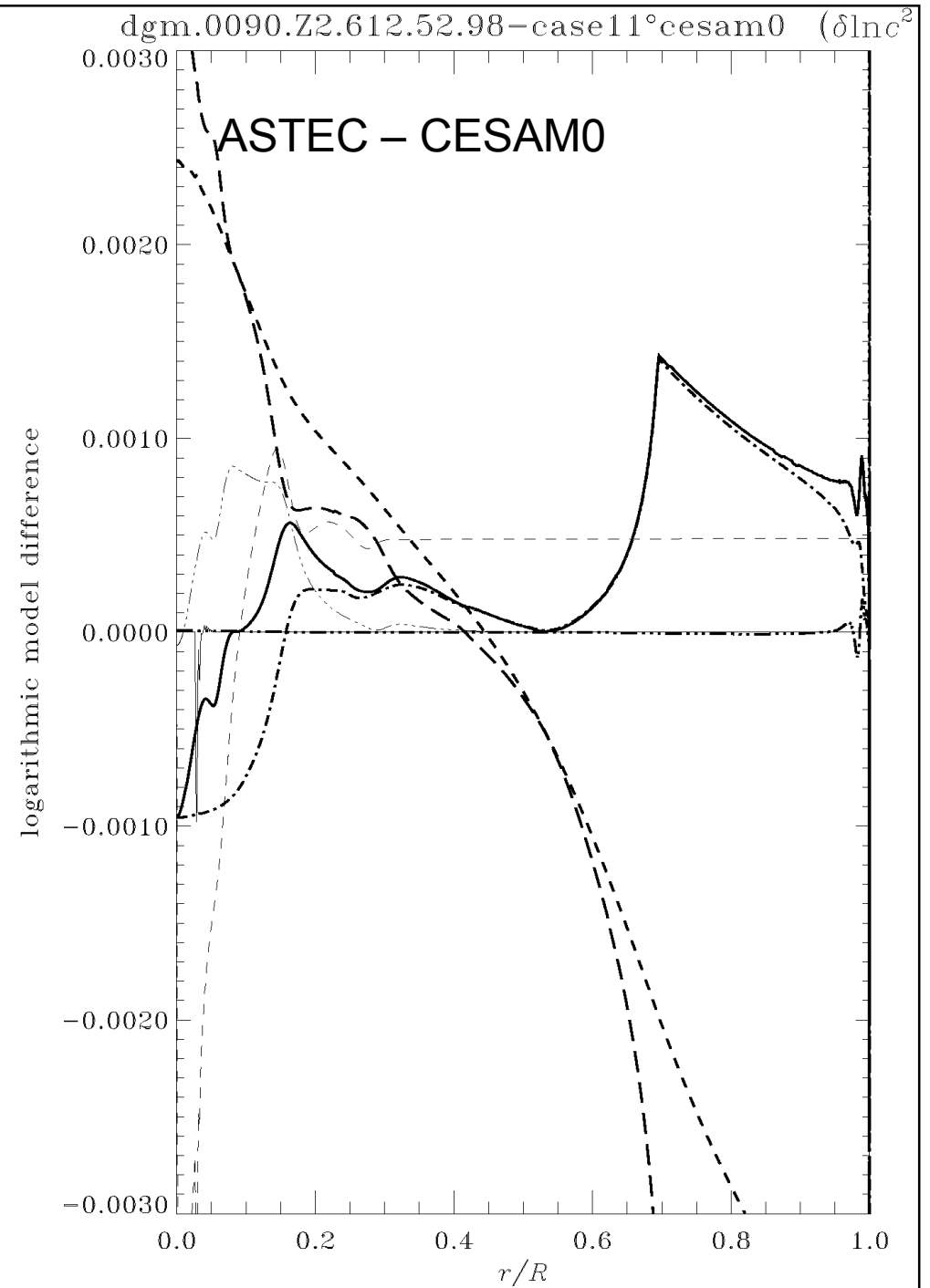
# Case 1.1

$0.9 M_{\odot}$ ,  $X_c = 0.35$

Line styles:

----- :  $\delta \ln T$   
----- :  $\delta \ln p$   
----- :  $\delta \ln \rho$   
————— :  $\delta \ln c^2$   
----- :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$   
----- :  $\delta \ln L$   
----- :  $\delta X$

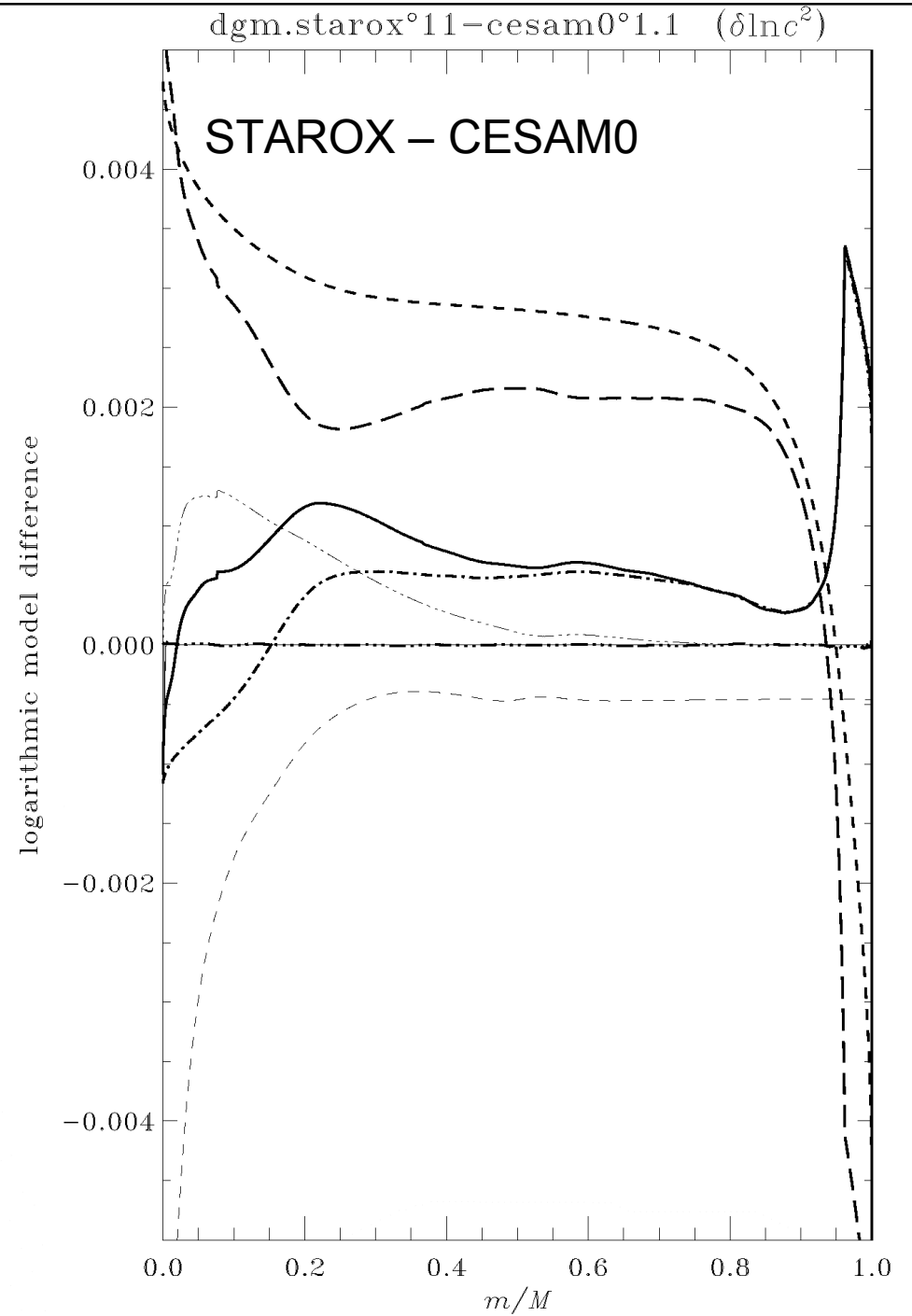


# Case 1.1

$0.9 M_{\odot}$ ,  $X_c = 0.35$

Line styles:

- |           |                         |           |                  |
|-----------|-------------------------|-----------|------------------|
| .....     | : $\delta \ln T$        | ————      | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | -----     | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | - · - · - | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |           |                  |
| .....     | : $\delta \ln \Gamma_1$ |           |                  |



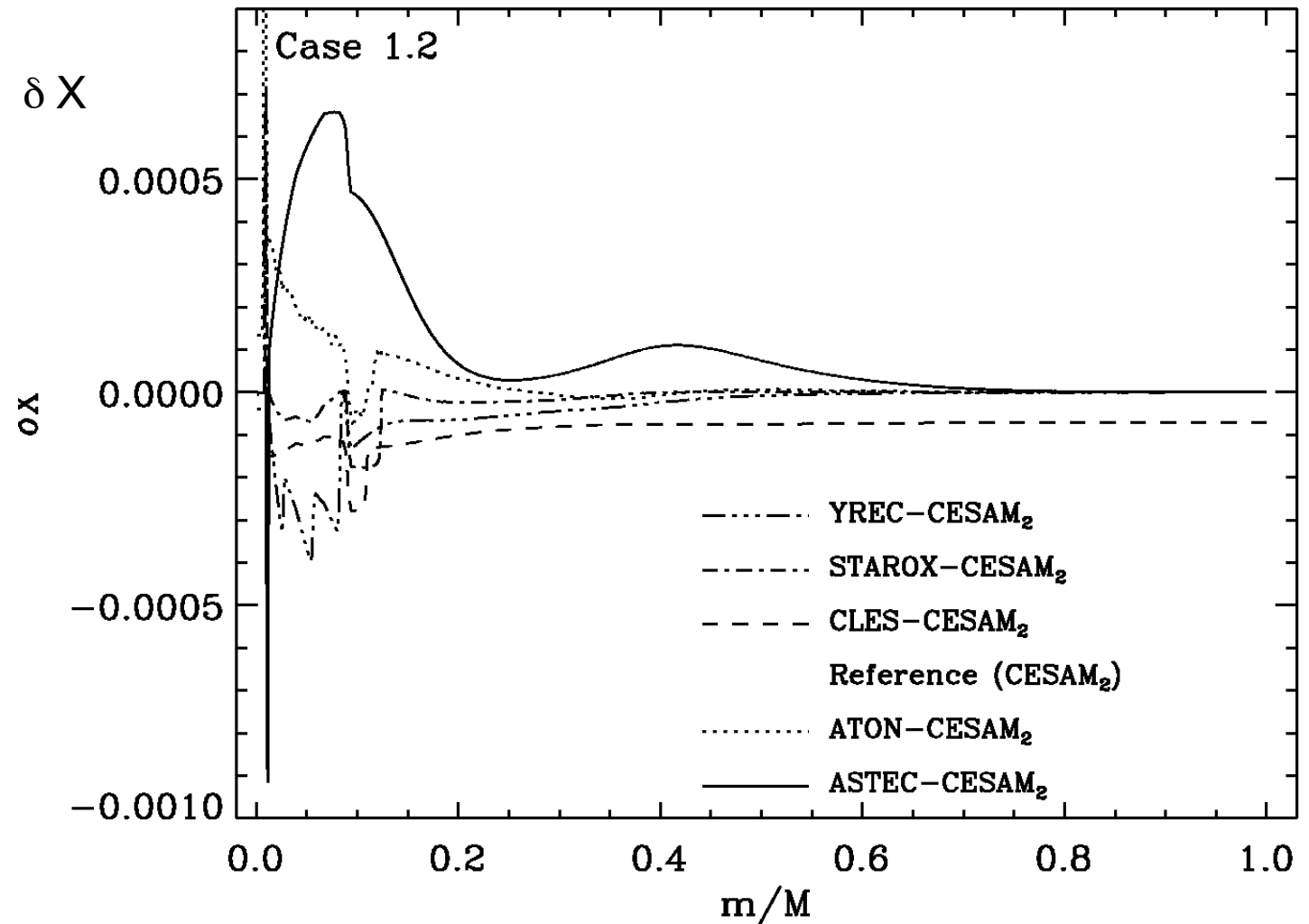
# Hydrogen abundance

1.2 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$



# Case 1.2

1.2 M-

$X_0 = 0.7$

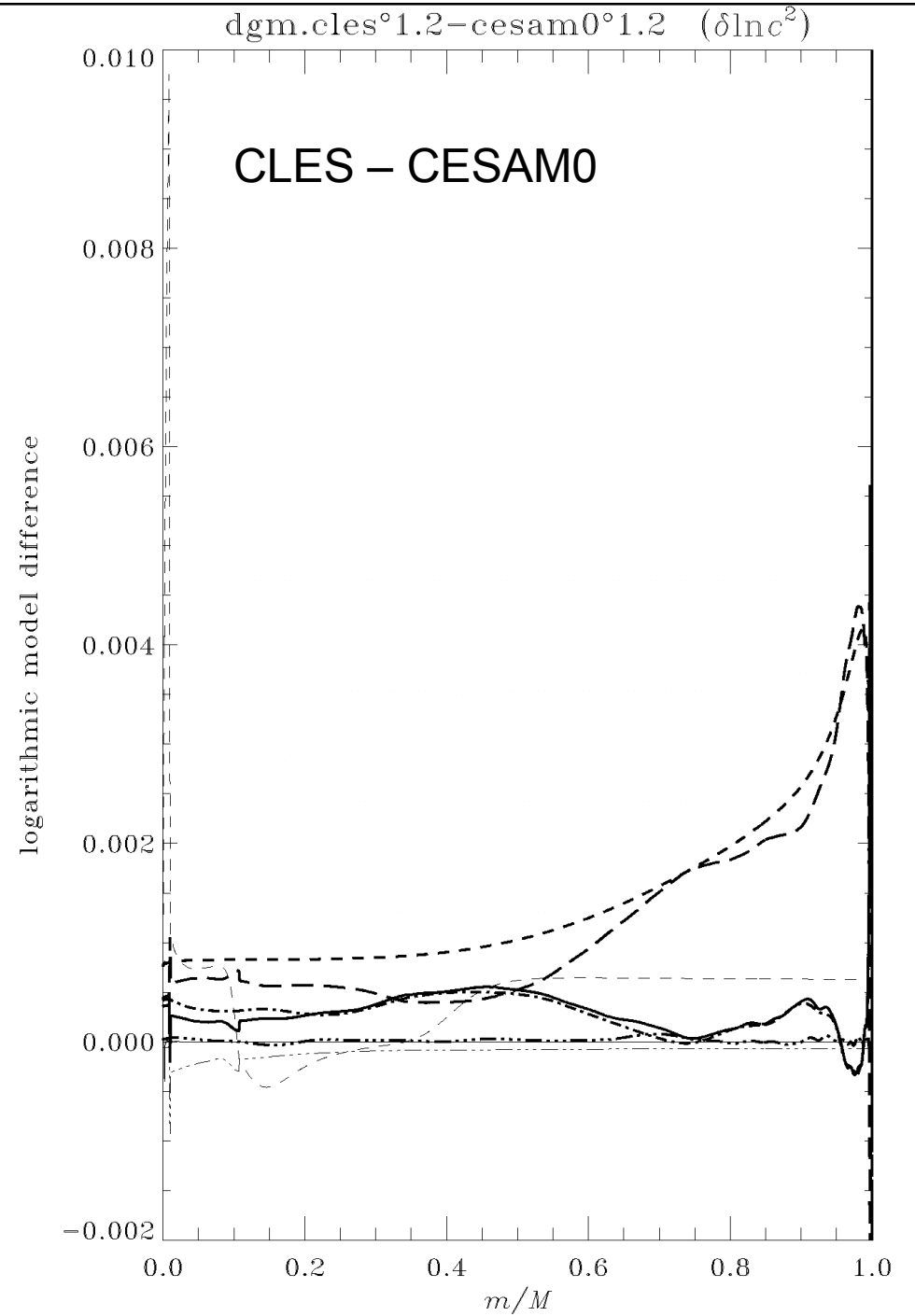
$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

- :  $\delta \ln T$
- :  $\delta \ln p$
- - - - - :  $\delta \ln \rho$
- :  $\delta \ln c^2$
- ..... :  $\delta \ln \Gamma_1$

- :  $\delta \ln q$
- :  $\delta \ln L$
- ..... :  $\delta X$



# Case 1.2

1.2 M-

$X_0 = 0.7$

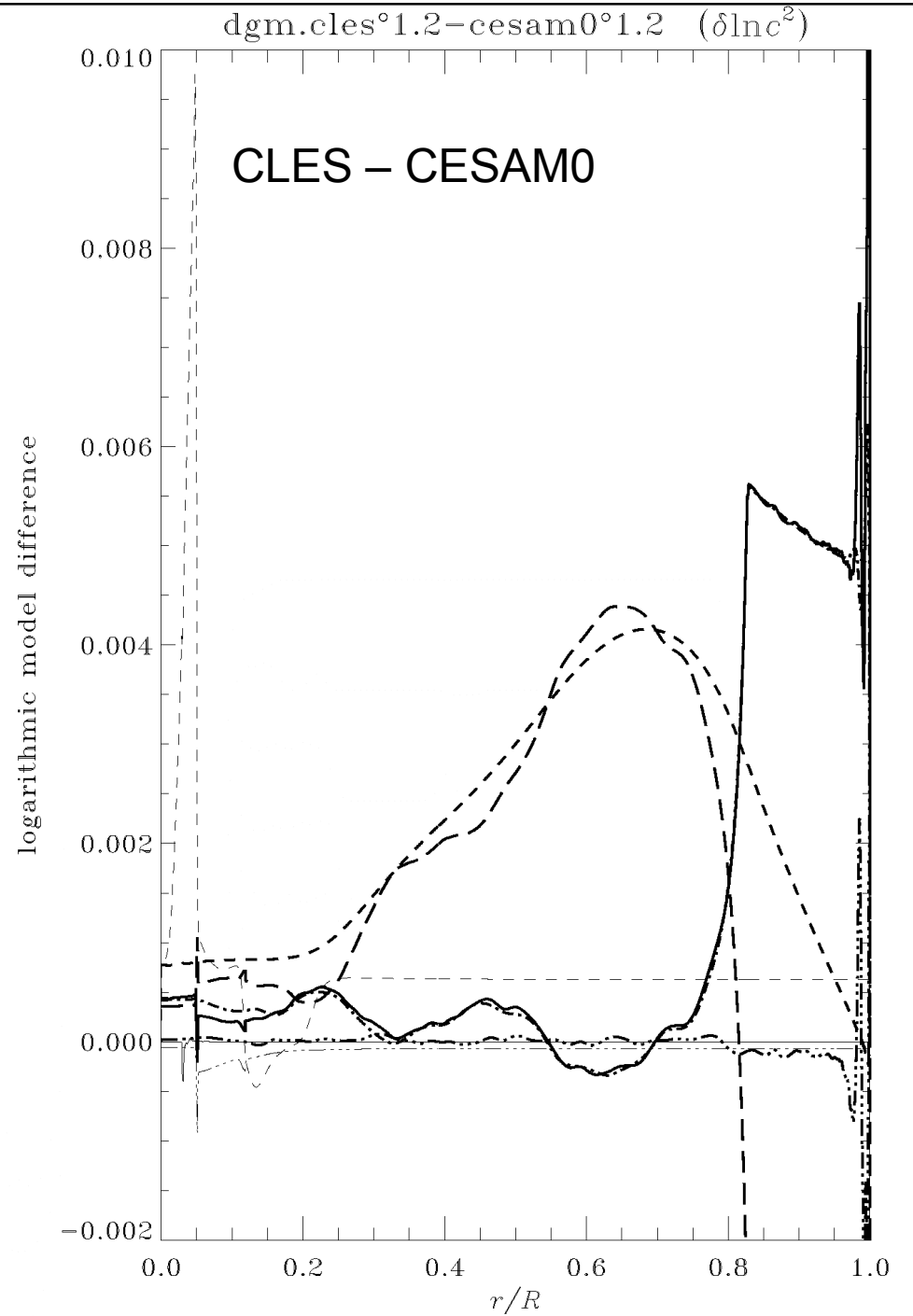
$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

- :  $\delta \ln T$
- :  $\delta \ln p$
- :  $\delta \ln \rho$
- :  $\delta \ln c^2$
- :  $\delta \ln \Gamma_1$

- :  $\delta \ln q$
- :  $\delta \ln L$
- :  $\delta X$





# Case 1.2

1.2 M-

$X_0 = 0.7$

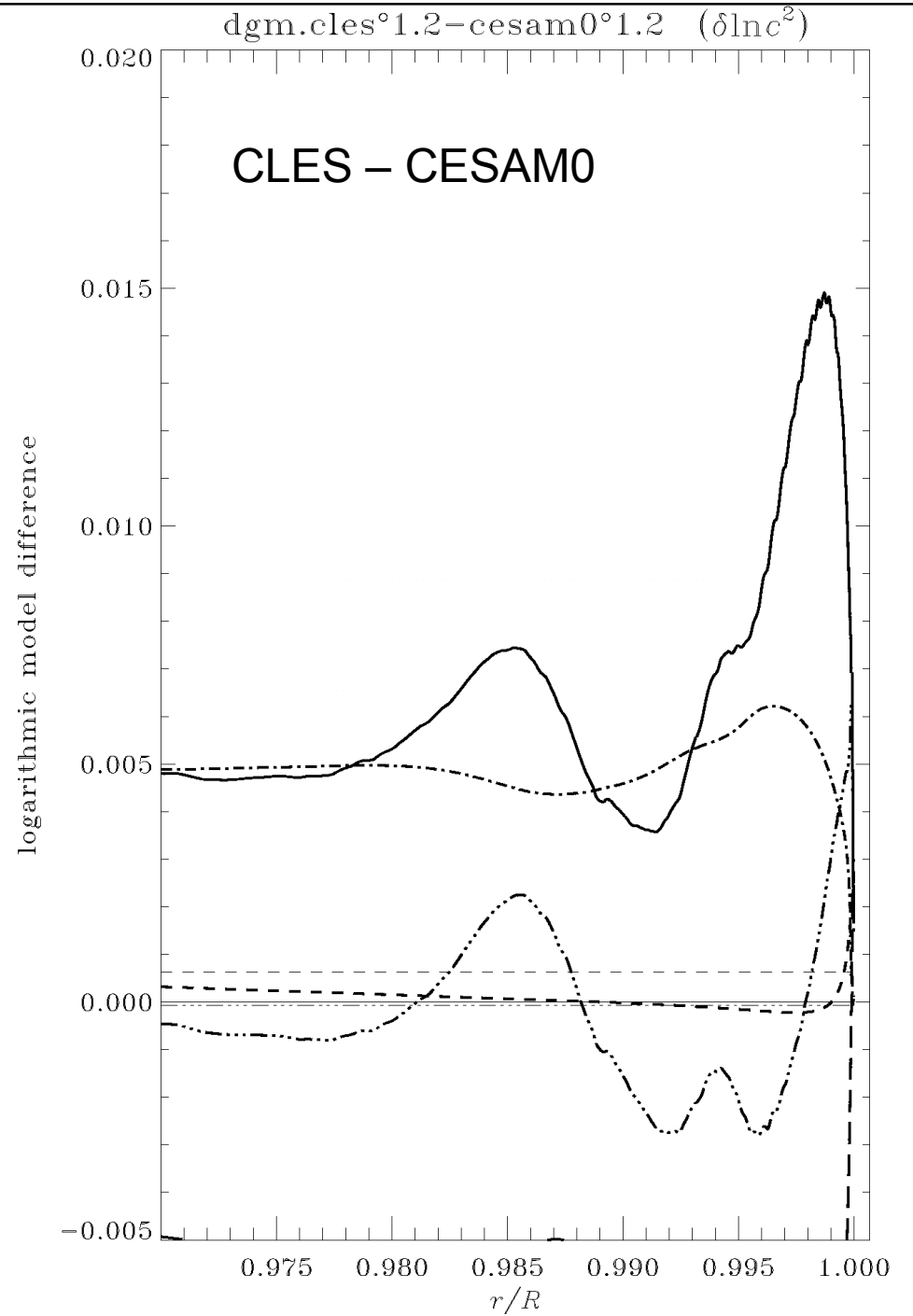
$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

..... :  $\delta \ln T$   
----- :  $\delta \ln p$   
- - - - - :  $\delta \ln \rho$   
————— :  $\delta \ln c^2$   
- · - · - :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$   
----- :  $\delta \ln L$   
- · - · - :  $\delta X$



# Case 1.2

1.2 M-

$X_0 = 0.7$

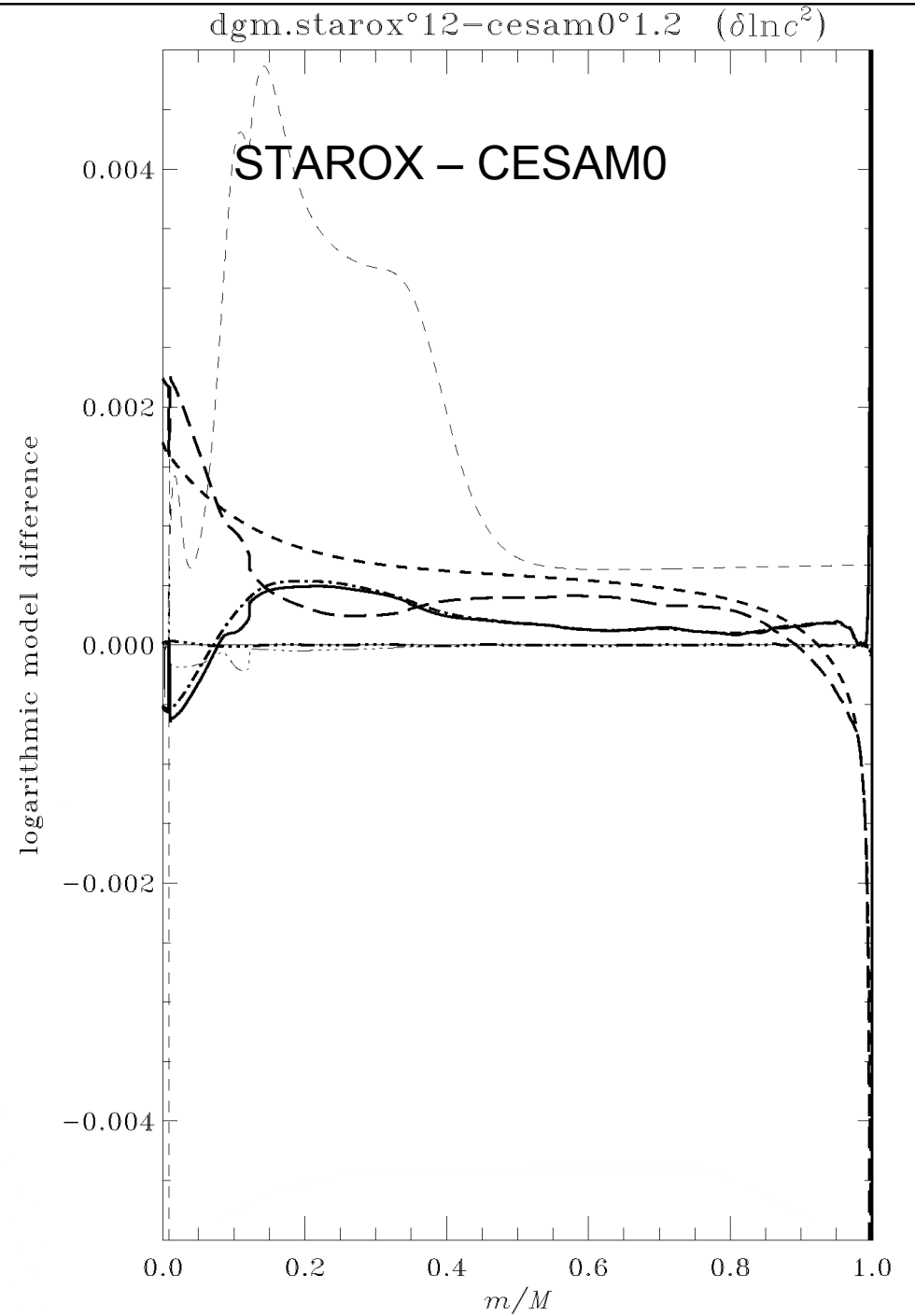
$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

..... :  $\delta \ln T$   
----- :  $\delta \ln p$   
- - - - - :  $\delta \ln \rho$   
————— :  $\delta \ln c^2$   
- · - · - :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$   
----- :  $\delta \ln L$   
- · - · - :  $\delta X$



# Case 1.2

1.2 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

..... :  $\delta \ln T$

----- :  $\delta \ln p$

- - - - - :  $\delta \ln \rho$

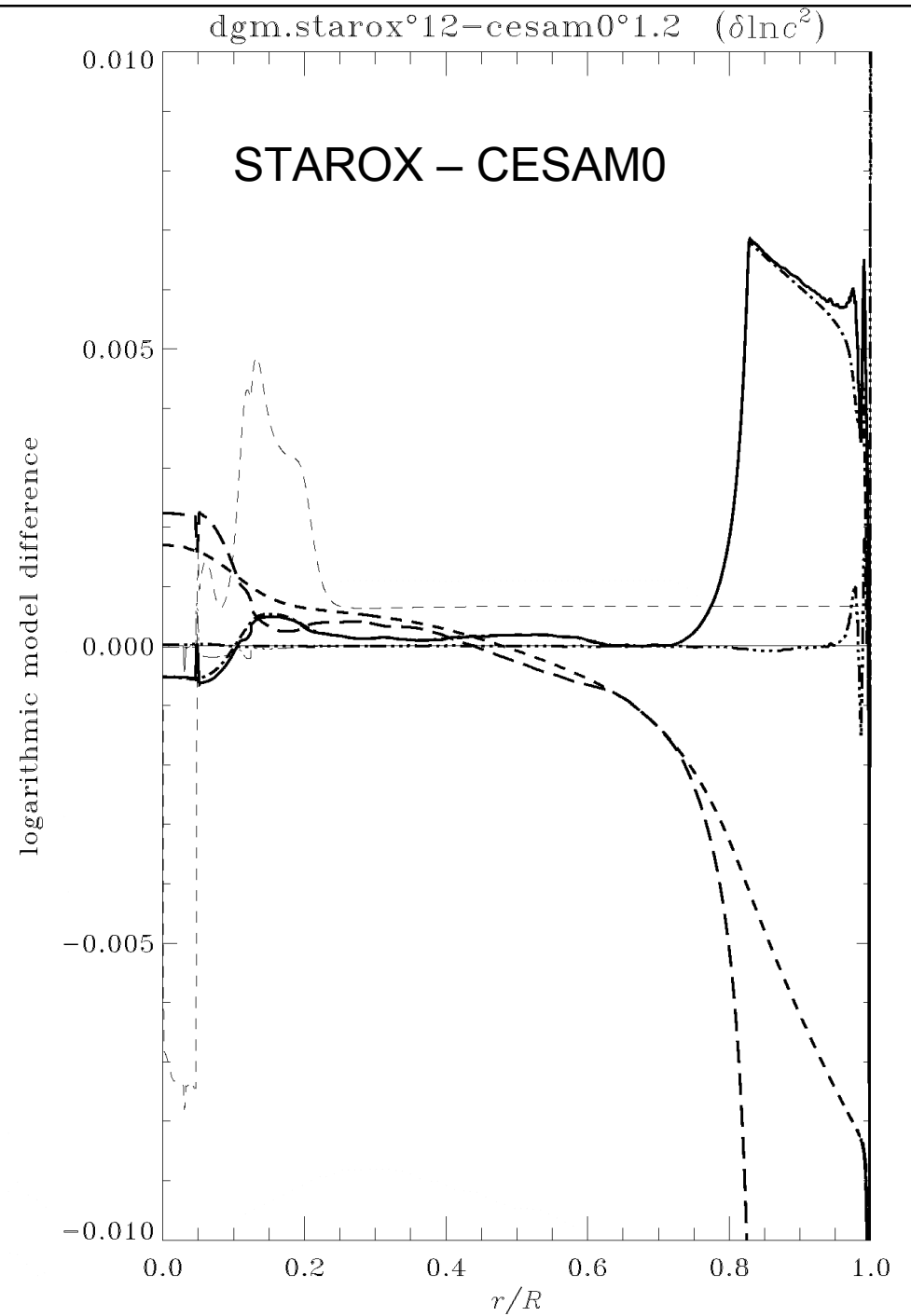
————— :  $\delta \ln c^2$

- · - · - · :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$

----- :  $\delta \ln L$

- · - · - · :  $\delta X$



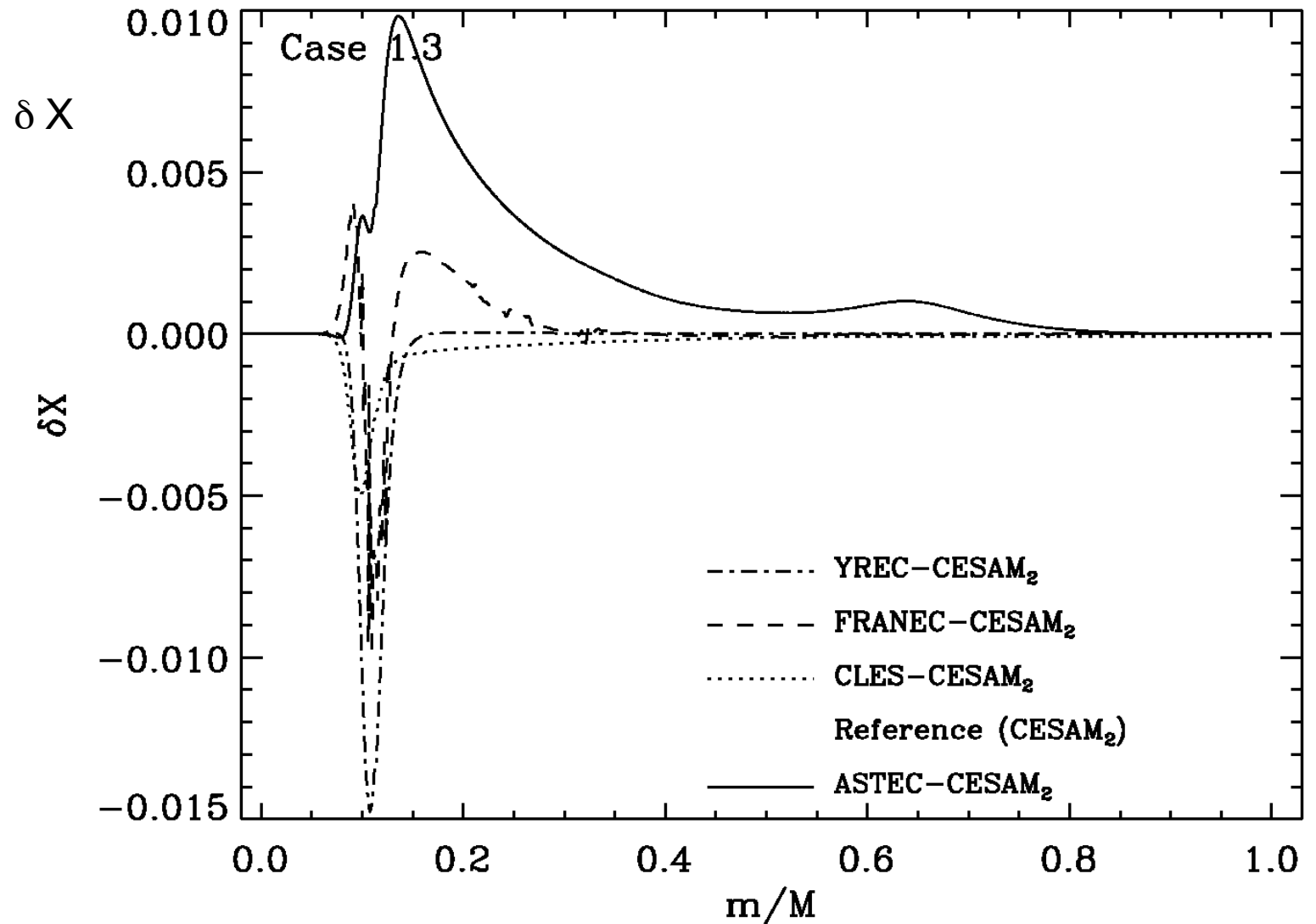
# Near-surface problems

- Differences in atmospheric treatment?
- Differences in mixing-length treatment?
- Results in different radii!

**Action: compare details of mixing-length formulations**

# Hydrogen abundance

1.2 M-  
 $X_0 = 0.73$   
 $Z_0 = 0.01$   
 $M_{\text{HeC}}/M = 0.1$



# Case 1.3

1.2 M-

$X_0 = 0.73$

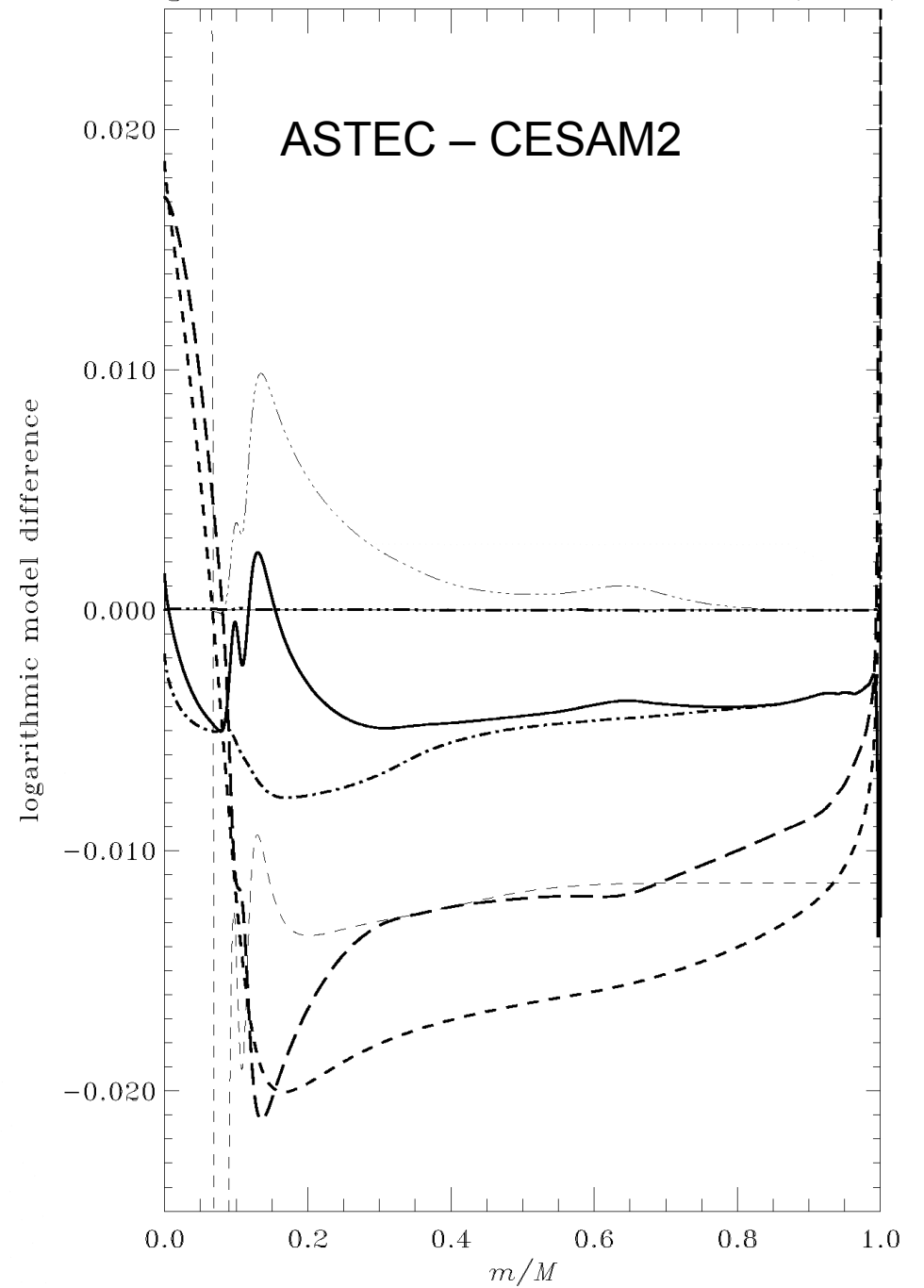
$Z_0 = 0.01$

$M_{\text{HeC}}/M = 0.1$

Line styles:

- ..... :  $\delta \ln T$
- :  $\delta \ln p$
- - - - - :  $\delta \ln \rho$
- :  $\delta \ln c^2$
- ..... :  $\delta \ln \Gamma_1$

- :  $\delta \ln q$
- :  $\delta \ln L$
- ..... :  $\delta X$



# Case 1.3

1.2 M-

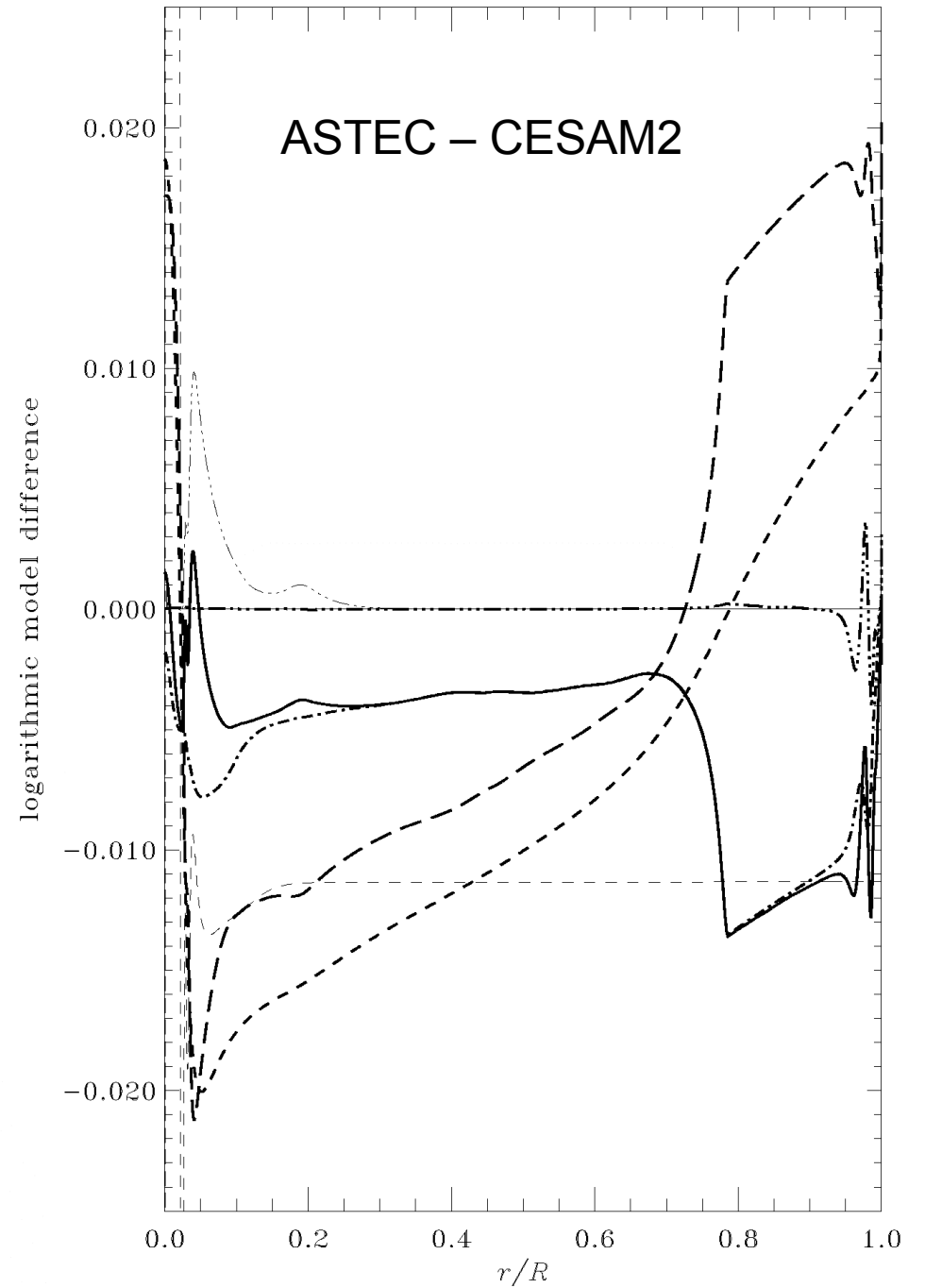
$X_0 = 0.73$

$Z_0 = 0.01$

$M_{\text{HeC}}/M = 0.1$

Line styles:

- |           |                         |           |                  |
|-----------|-------------------------|-----------|------------------|
| .....     | : $\delta \ln T$        | ————      | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | - - - - - | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | .....     | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |           |                  |
| .....     | : $\delta \ln \Gamma_1$ |           |                  |



# Hydrogen abundance

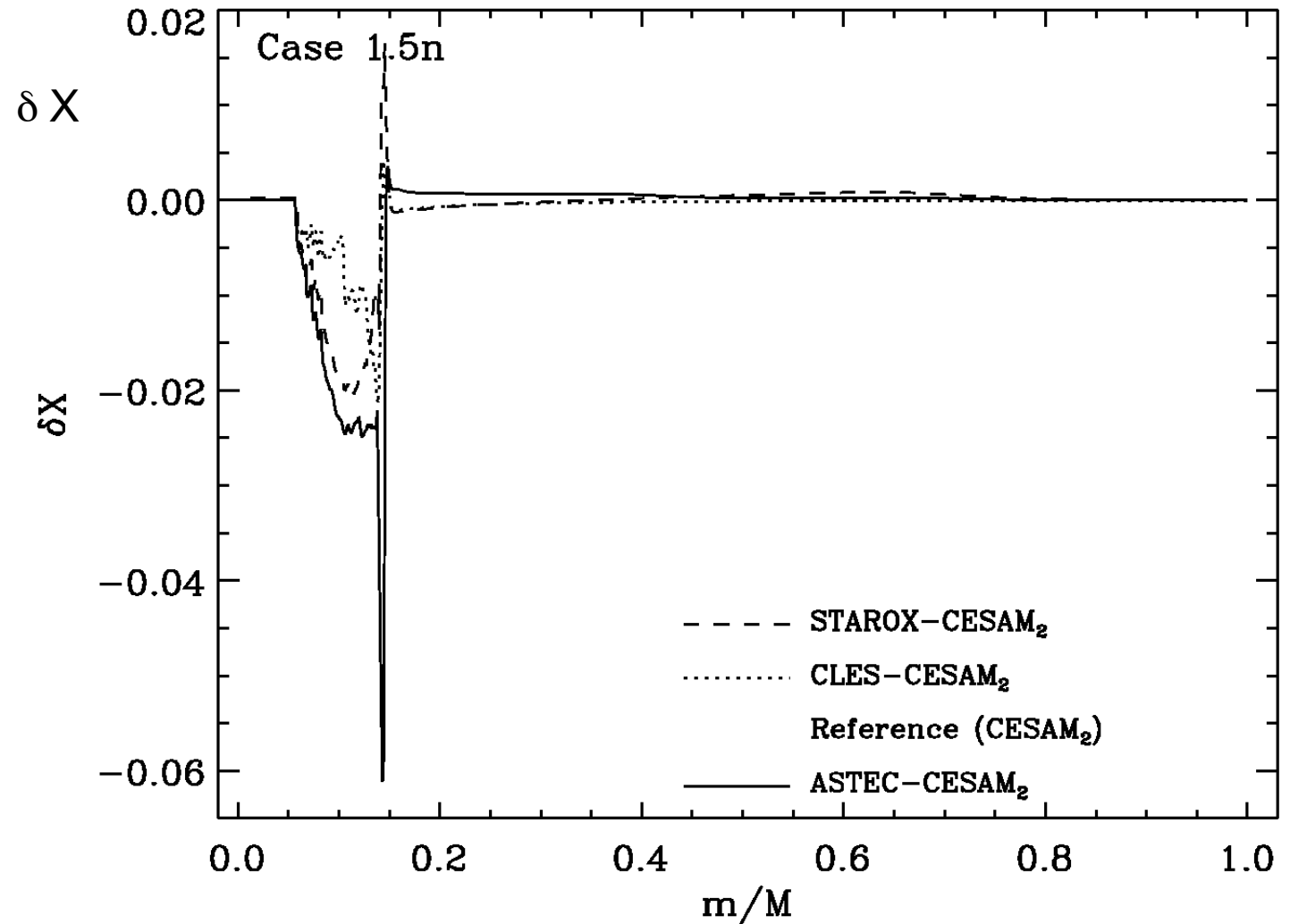
2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

No overshoot





# Case 1.5n

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

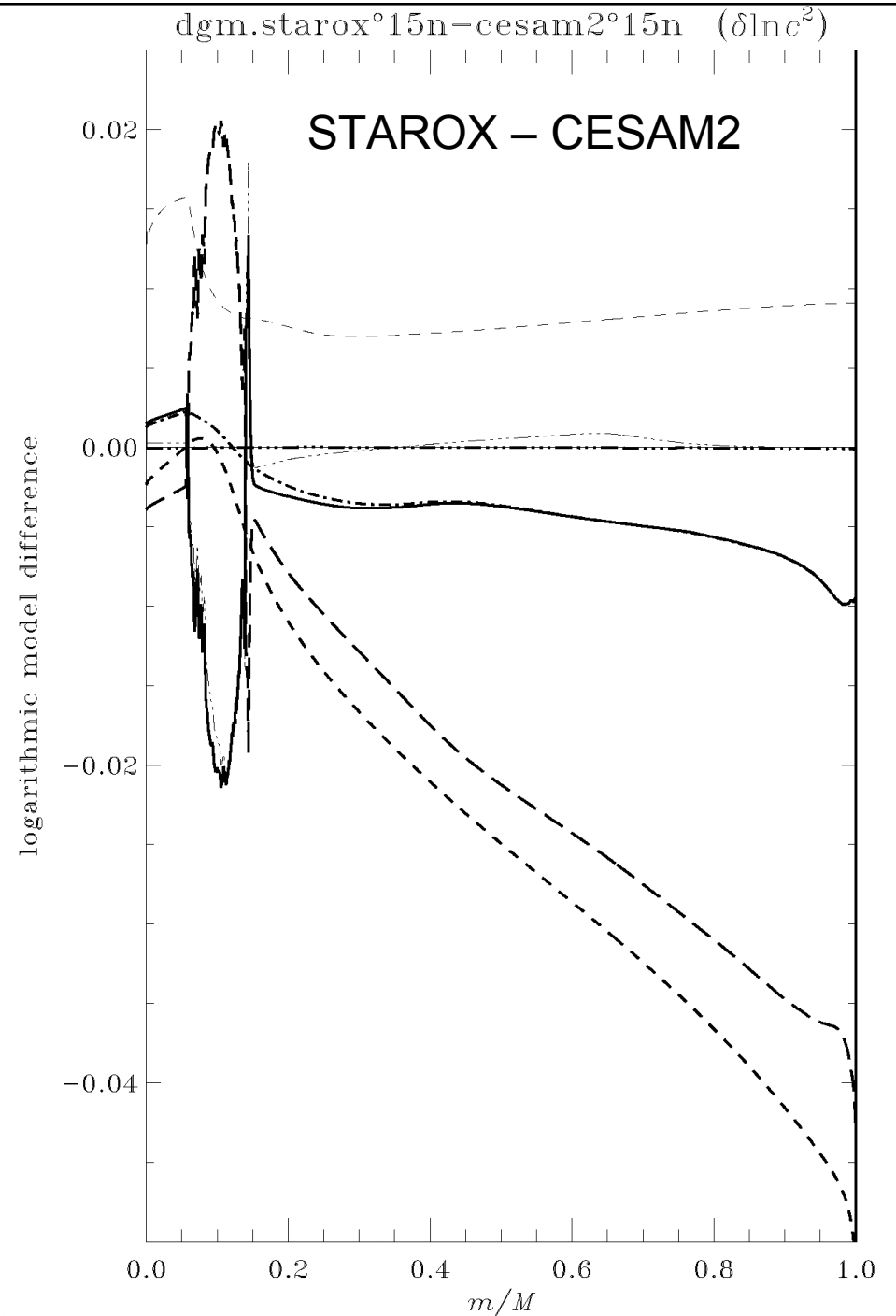
$X_c = 0.01$

No overshoot

Line styles:

..... :  $\delta \ln T$   
----- :  $\delta \ln p$   
- - - - - :  $\delta \ln \rho$   
————— :  $\delta \ln c^2$   
- · - · - :  $\delta \ln \Gamma_1$

————— :  $\delta \ln q$   
----- :  $\delta \ln L$   
- · - · - :  $\delta X$



# Case 1.5n

2.0 M-

$X_0 = 0.72$

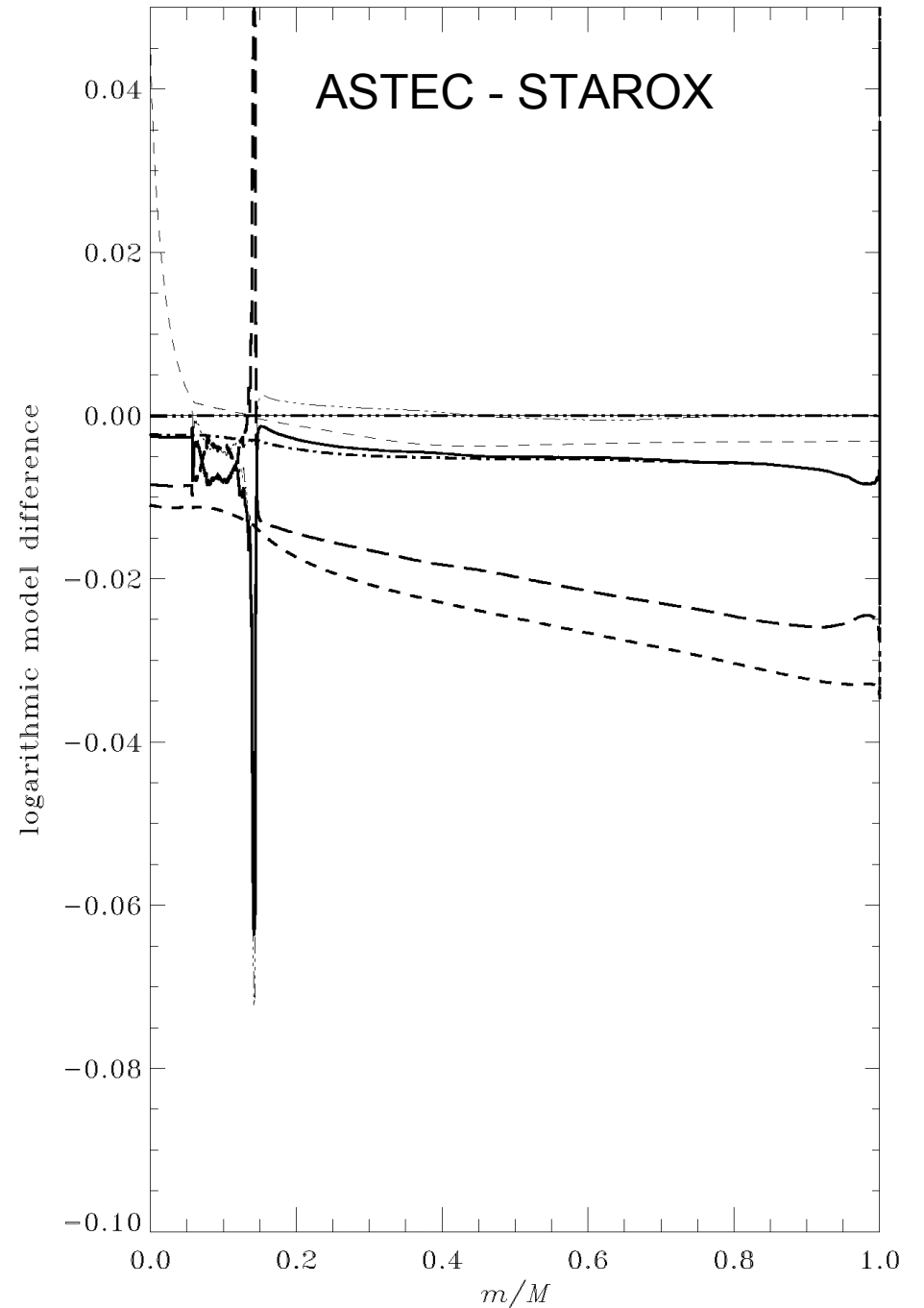
$Z_0 = 0.02$

$X_c = 0.01$

No overshoot

Line styles:

- |           |                         |       |                  |
|-----------|-------------------------|-------|------------------|
| .....     | : $\delta \ln T$        | ————  | : $\delta \ln q$ |
| -----     | : $\delta \ln p$        | ----- | : $\delta \ln L$ |
| - - - - - | : $\delta \ln \rho$     | ..... | : $\delta X$     |
| ————      | : $\delta \ln c^2$      |       |                  |
| .....     | : $\delta \ln \Gamma_1$ |       |                  |



# Hydrogen abundance

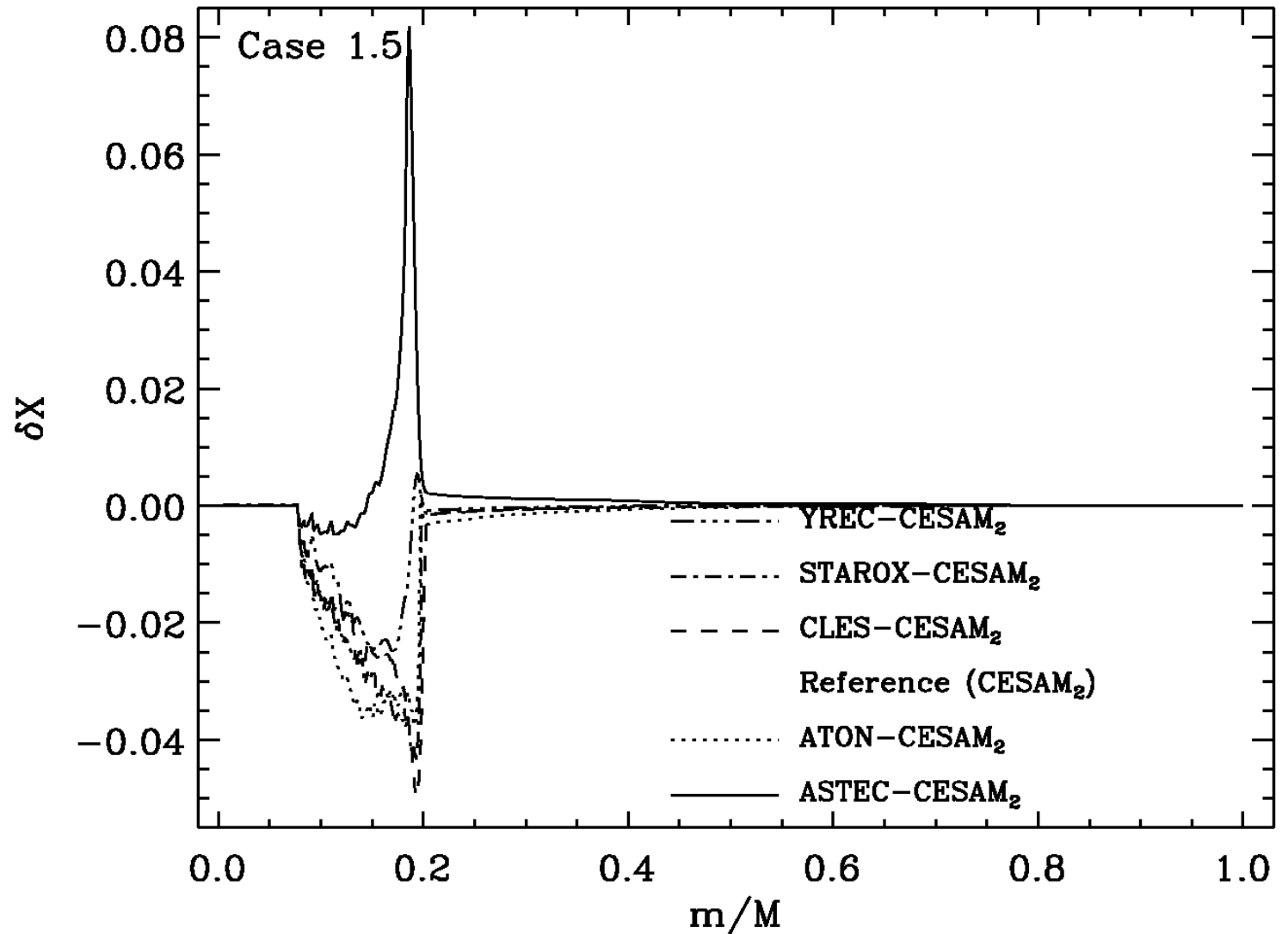
2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

Overshoot,  
 $0.15 H_p$



# Case 1.5

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

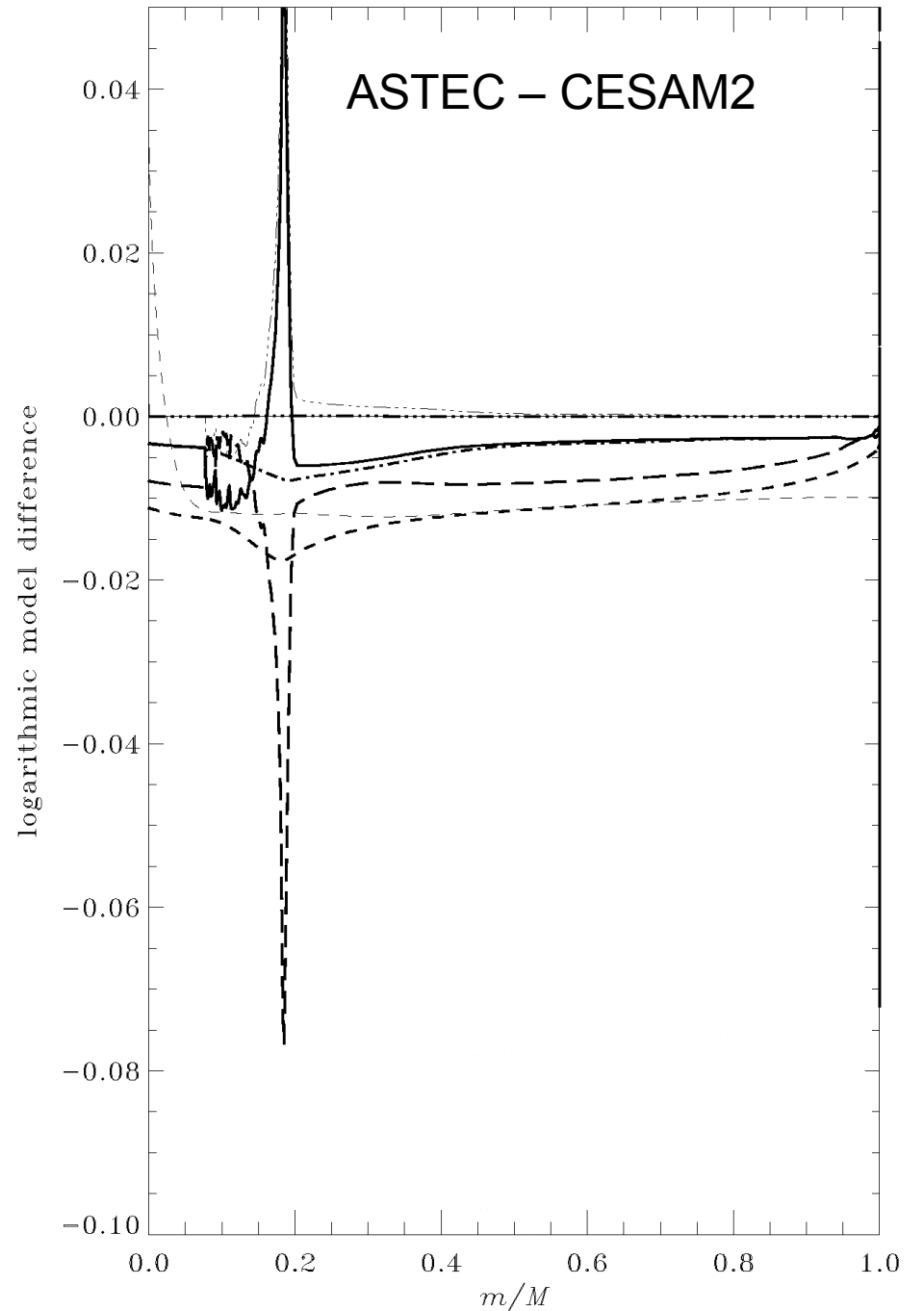
$X_c = 0.01$

Overshoot  
0.15  $H_p$

Line styles:

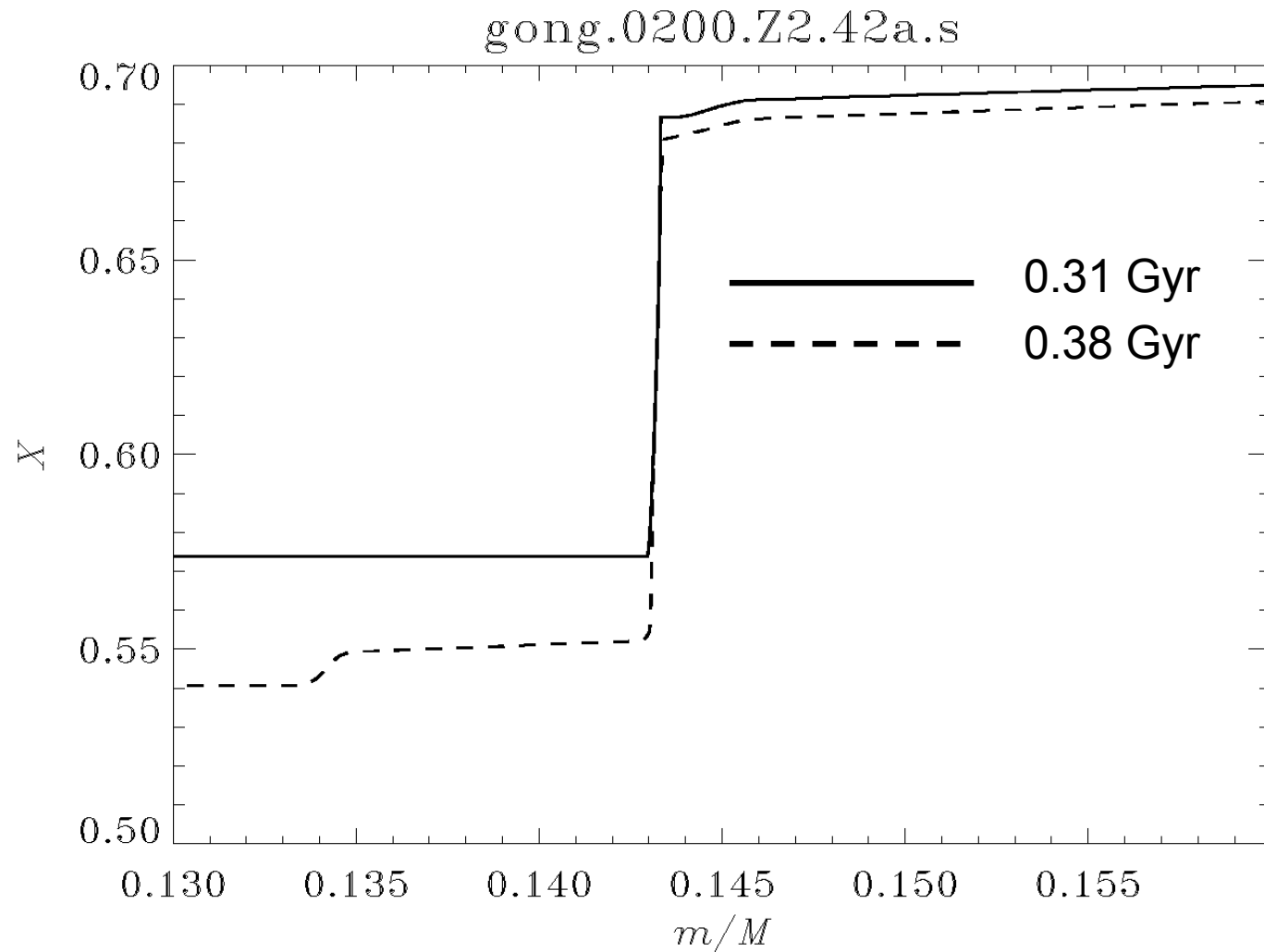
- ..... :  $\delta \ln T$
- :  $\delta \ln p$
- - - - - :  $\delta \ln \rho$
- :  $\delta \ln c^2$
- ..... :  $\delta \ln \Gamma_1$

- :  $\delta \ln q$
- :  $\delta \ln L$
- ..... :  $\delta X$

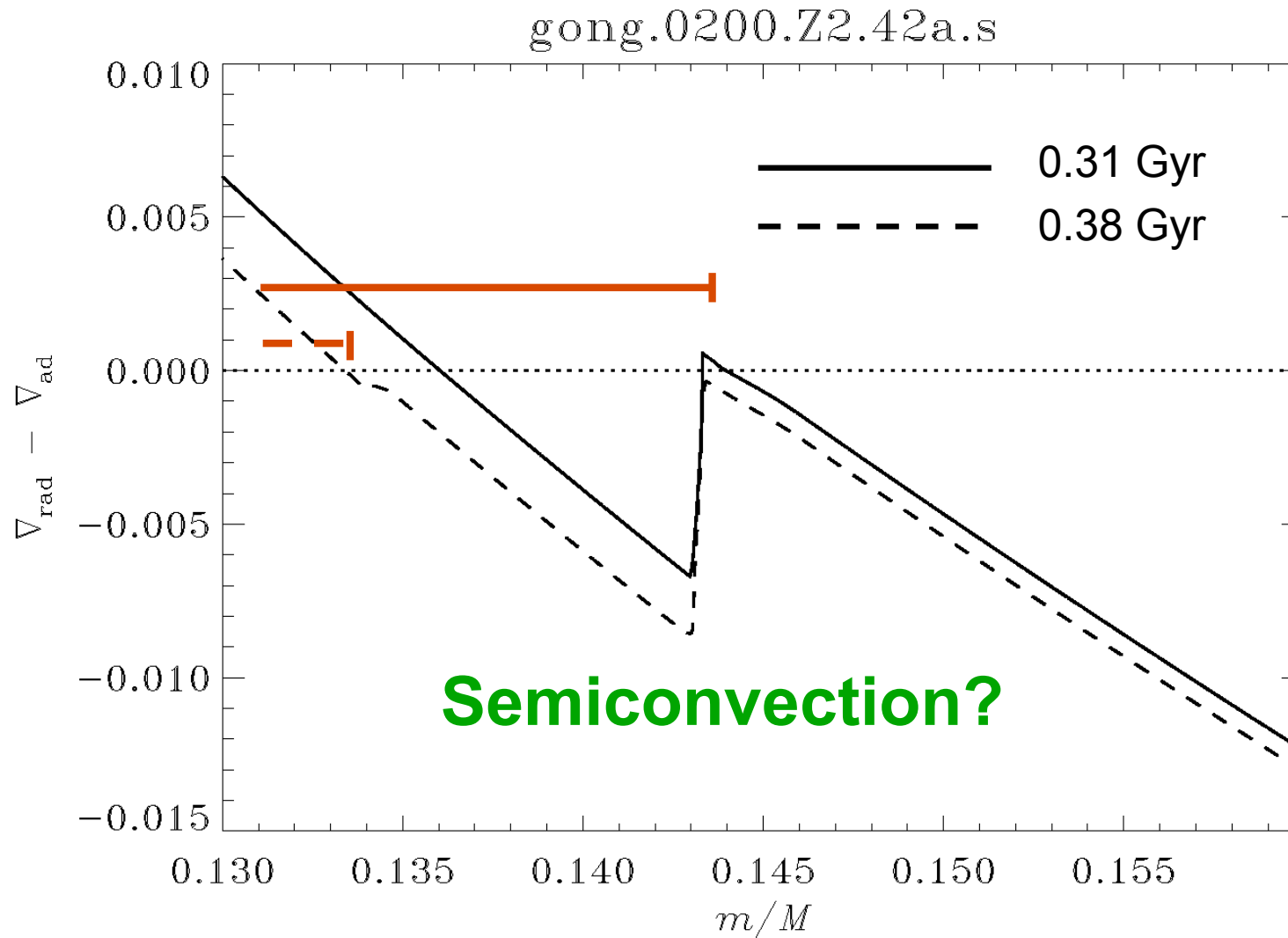


# Problems with growing convective core

2 M-  
Z = 0.02



# Problems with growing convective core



# Semiconvection

