ATON code for stellar evolution

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Some history …

- At the end of the 1970's, Italo Mazzitelli (IAS) writes the stellar evolution code ATON.
- In 1991 a new convective model (FST) is implemented in the code, as an alternative to the mixing length theory (MLT).
- In 1998 the diffusive scheme to treat simultaneously nuclear burning and mixing of chemicals is included.
- In 2000 Luiz Temistoklis Mendez (University of Belo Horizonte) introduced an algorithm to take into account the structural effects due to rotation.
- At the same time, Josefina Montalbàn (University of Liegi) implemented into ATON a routine that uses non grey boundary conditions.

ATON code can follow all the evolutionary phases from the pre-MS up to Carbon ignition.

The 4 structural equations, describing hydrostatic equilibrium, conservation of mass and energy, and the modality of energy transport, are integrated, keeping fixed the chemical composition, with a Newton – Raphson scheme.

The code allows three starting modalities: pre-MS, ZAMS, and HB.

The independent variable is the Mass throughout the star. Dependent variables are radius, pressure, temperature and luminosity.

The internal integration of the 4 structural equations can be performed from the centre to the outermost layers with two different BCs

Grey treatment

- The bottom of the optical atmosphere is assumed to be at $\tau = 2/3$
- Values of pressure and temperature at the matching layer are obtained by integrating the hydrostatic equilibrium equation and a simple T(tau) relation.

Non Grey treatment

- The matching point can be chosen in the range $1 < \tau < 100$
- Pressure and temperature at the matching layer are obtained by interpolating among the atmospheres provided for various Teff's and gravities by

1) MLT Heiter et al. (2002)
   (Teff > 4000 K)
2) FST Heiter et al. (2002)
   (Teff > 4000 K)
3) MLT BCAH (1997)
   (Teff > 3000 K)

Some numerical details …

- To avoid numerical problems close to the surface, where we may have

$$\Delta M / M \approx 10^{-12} - 10^{-18}$$

the code performs numerical derivatives directly on the ΔM vector. This also allows straightforward computation of the gravo-thermal energy up to the stellar surface.

- Internal zoning of the structure is reassessed at each physical time step, with particular care to central and surface regions, burning shells, convective borders, and overadiabaticity zones.

- Pre-MS: 1500 mesh points
- Red giants: 2000 mesh points
- HB: 2000-2500 mesh points
- AGB: 2500-5000 mesh points

- The physical time-step is evaluated allowing maximum variations of both local and integrated quantities, with special care to the single luminosities (CNO, pp, He-burning..), the variation of central abundances, and the maximum variation of the 4 dependent variables.

- Sun PMS: 1000 time steps
- Sun He-flash: 15000-2000 time steps
- HB evolution: 2000-3000 time steps
- AGB TPs: 5000 time steps (2 following pulses)

For chemical evolution, we adopt the implicit scheme by Arnett & Truram (1969).

It can be considered as a first-order Runge Kutta, so that the integration error is $\propto (\Delta X)^2$

Any physical time step, for the purpose of chemical evolution only, is further divided into at least 10 chemical time steps.

Any mechanism leading to chemical abundances variation (e.g. nuclear burning) is repeated for all the chemical time steps.

Convection

The stability of any layer against convective motions is extablished on the basis of the classic Schwartzschild criterium.

Within convective regions, the temperature gradient is found via some local approximations, aimed at finding out the overadiabaticity.

Within a completely local framework, the value of overadiabaticity can be found either via the traditional MLT, or by using the FST convective model.

MLT vs FST

MLT

The convective eddies spectrum is approximated by a Dirac function peaked around L.

The eddies are assumed to travel for a typical distance Λ

$$\Lambda = l = \alpha H_p$$

FST

All the eddies' dimensions are taken into account.

The mixing length is assumed to be

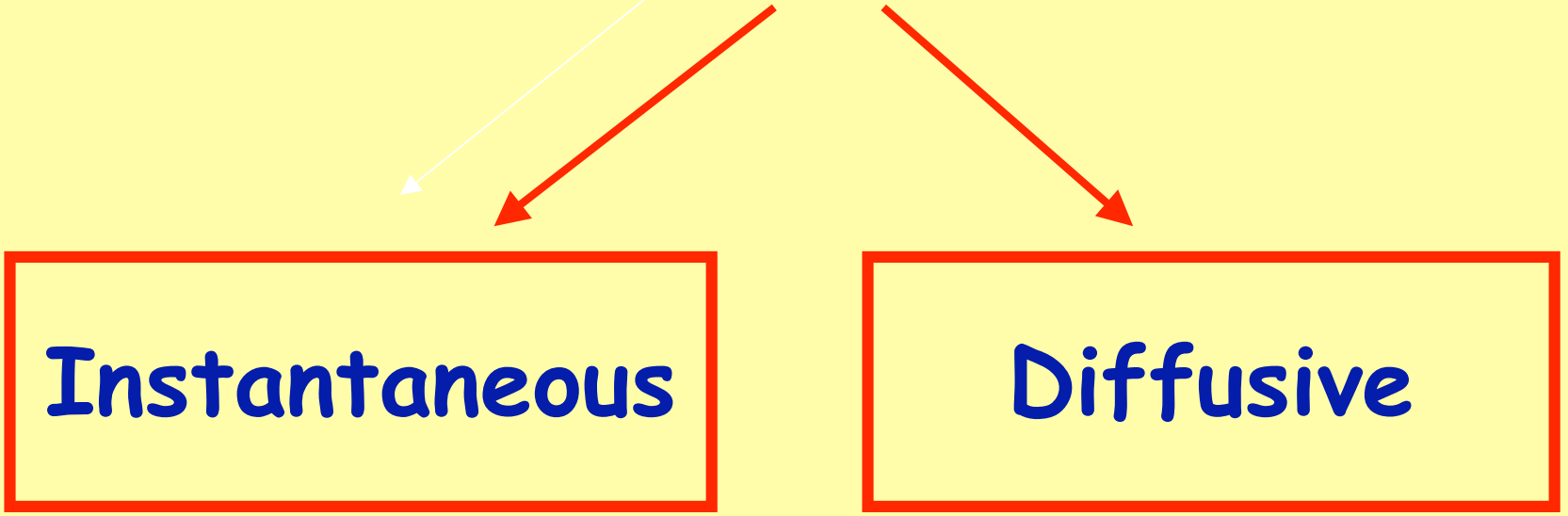
$$\Lambda = z + \beta H_p$$

Within highly efficient convective regions the FST fluxes are intrinsecally larger by a factor of 10.

Mixing & Burning

After the Newton – Raphson has reached convergency, the chemical composition is allowed to change due both to mixing of chemicals (convective regions) and nuclear reactions.

To deal with convective regions that are nuclearly active, we have the possibility of following two different approaches:



Instantaneous mixing

The process of mixing is assumed to be much faster than any nuclear reaction, so that any convective region is assumed to be fully homogenized.

We may consider overshooting by allowing mixing of chemicals beyond the formal borders for a distance

$$l_{OV} = \nu H_p$$

Average chemistry and reaction rates are evaluated in the whole convective region, and the linearization procedure is applied, as it was a single mesh-point

Diffusive mixing

According to first principles, in the presence of both nuclear reactions and turbulent mixing, the local variation of any element follows the diffusion equation

$$\left(\frac{dX_i}{dt}\right) = \left(\frac{\partial X_i}{\partial t}\right)_{nucl} + \frac{\partial}{\partial m_r}\left[(4\pi r^2 \rho)^2 D \frac{\partial X_i}{\partial t}\right]$$

Local approximation 

$$D = \frac{1}{3} v \Lambda$$

In solving the diffusive equation, our choice was to expand the nuclear term as a function of local abundances and cross-sections, and solve it together with the diffusion matrix for the whole convective region.

The diffusive approach leads to a new formulation of overshooting, described as an exponential decay of turbulent velocity beyond the formal borders of the instability regions:

$$v = v_b \exp\left(\frac{1}{\zeta} \ln \frac{P}{P_b}\right)$$