

Summary of Aarhus workshop

Some selected results

Intrinsic numerical accuracy

- Compare models computed with a given code and given parameters
- Vary number of meshpoints
- Vary number of timesteps

Case 1.1

0.9 M₋, $X_c = 0.35$

³He in equilibrium

Test effect of no. of meshpoints:
(N = 1200) – (N = 600)

Line styles:

..... : $\delta \ln T$

----- : $\delta \ln p$

- - - - - : $\delta \ln \rho$

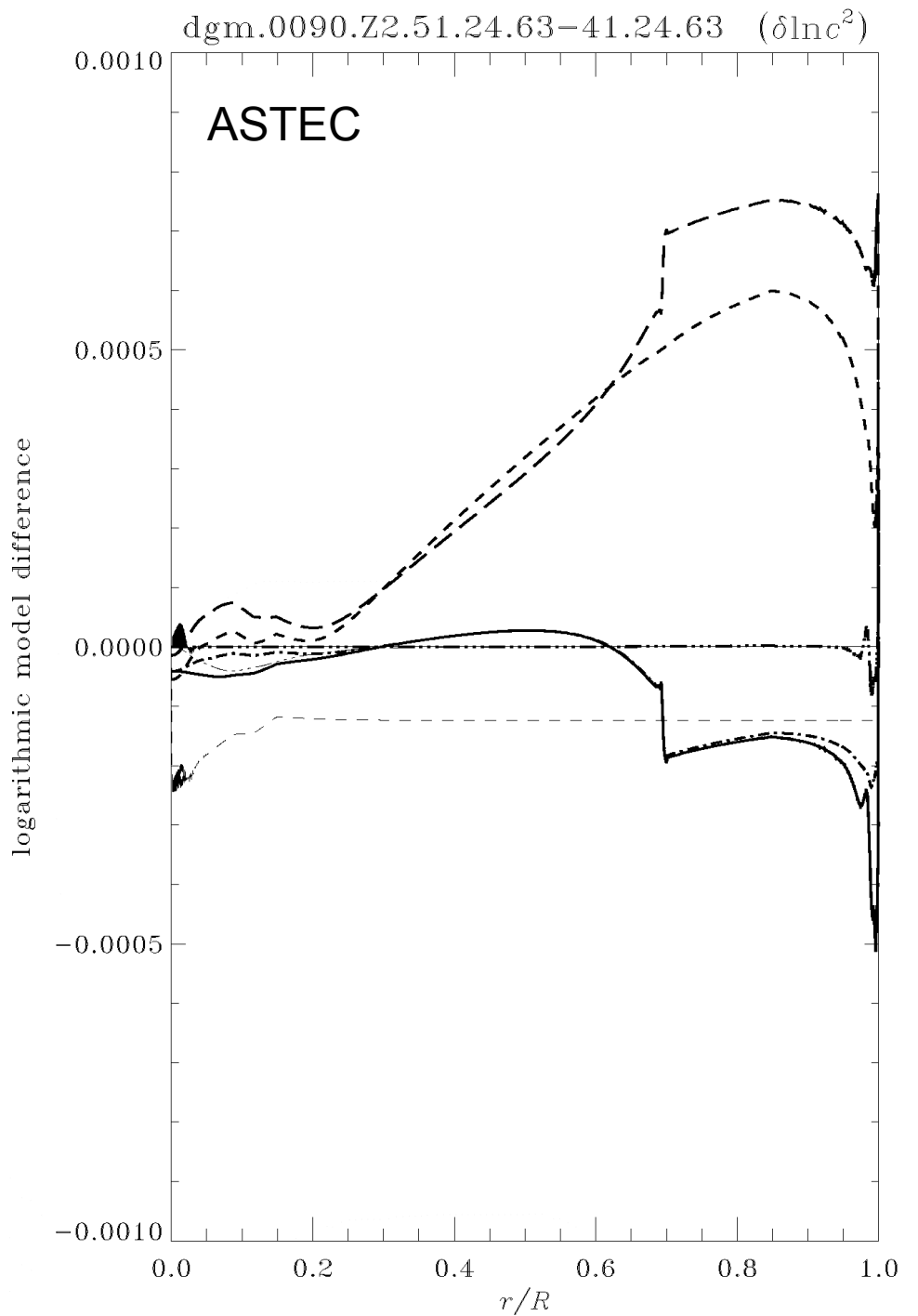
———— : $\delta \ln c^2$

- · - · - · : $\delta \ln \Gamma_1$

———— : $\delta \ln q$

----- : $\delta \ln L$

- · - · - · : δX



Case 1.1

0.9 M_⊙, $X_c = 0.35$

³He in equilibrium

Test effect of no. timesteps:

($N_t = 24$) – ($N_t = 13$)

($\Delta y_{\max} = 0.025$) – ($\Delta y_{\max} = 0.05$)

Line styles:

..... : $\delta \ln T$

----- : $\delta \ln p$

----- : $\delta \ln \rho$

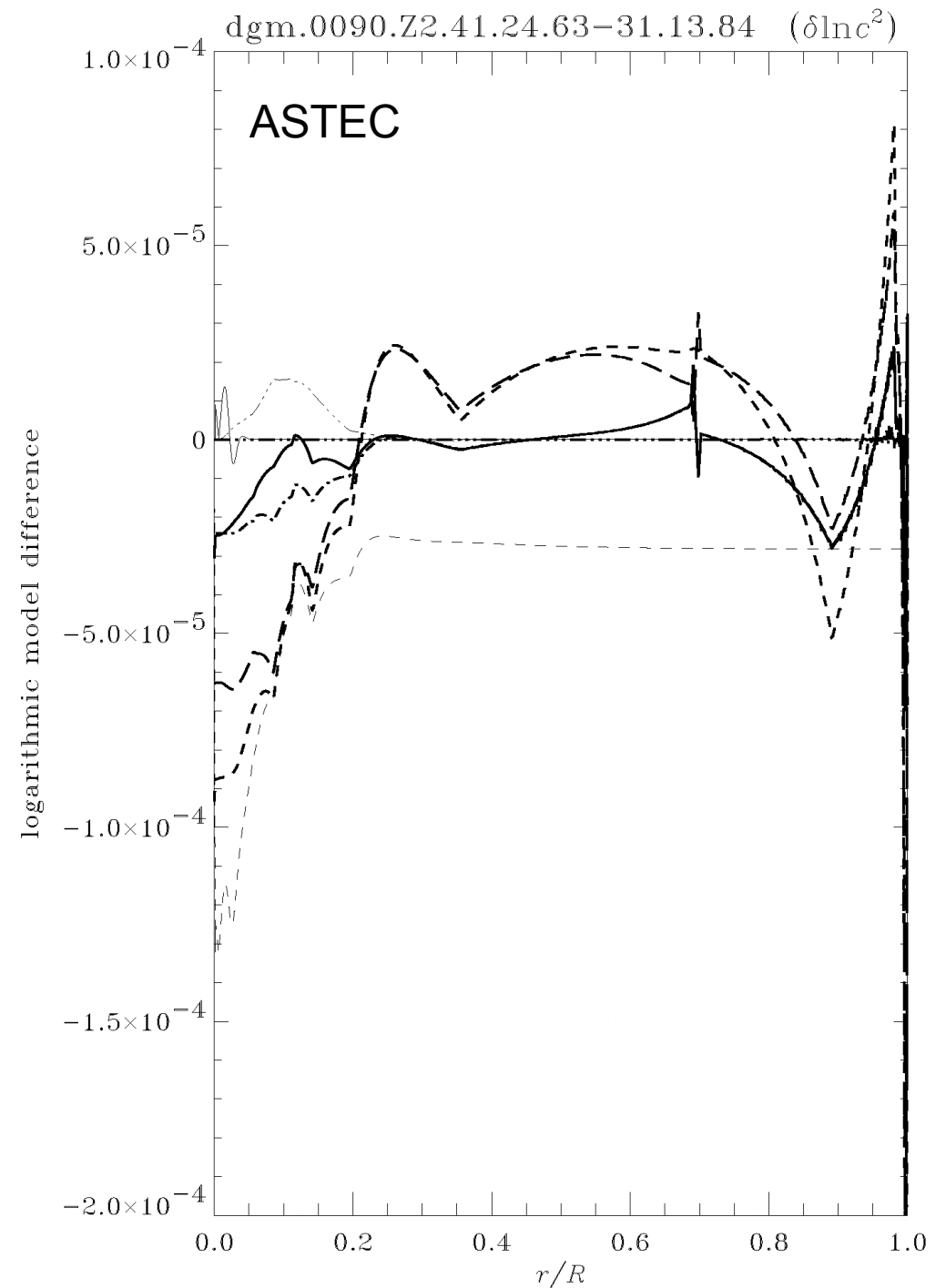
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Case 1.3

1.2 M_⊙, M_c = 0.1 M_⊙

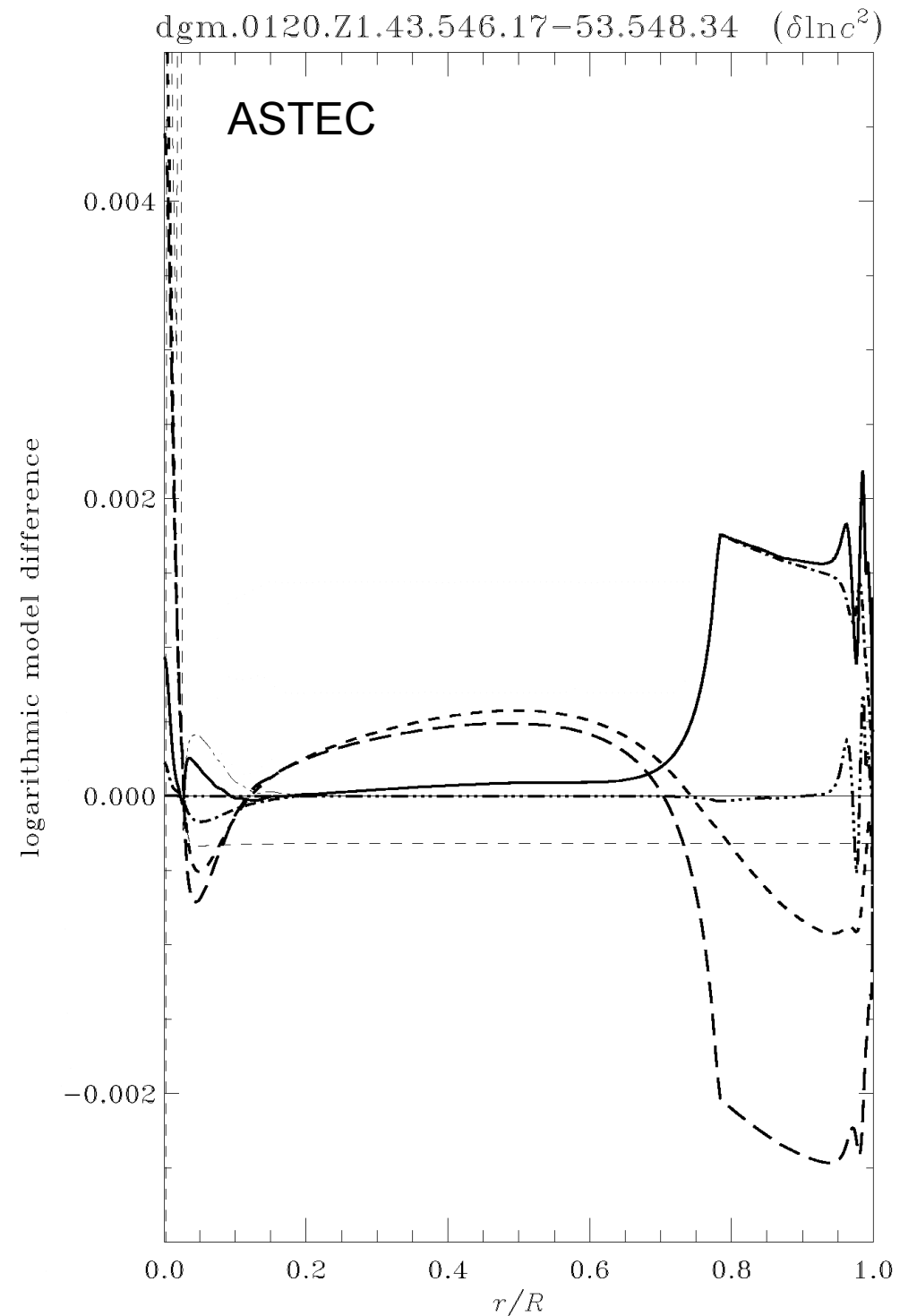
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Test effect of no. of meshpoints:
(N = 600) – (N = 1200)

Line styles:

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----- : $\delta \ln p$
----- : $\delta \ln \rho$
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1.2 M_{\odot} , $M_c = 0.1 M_{\odot}$

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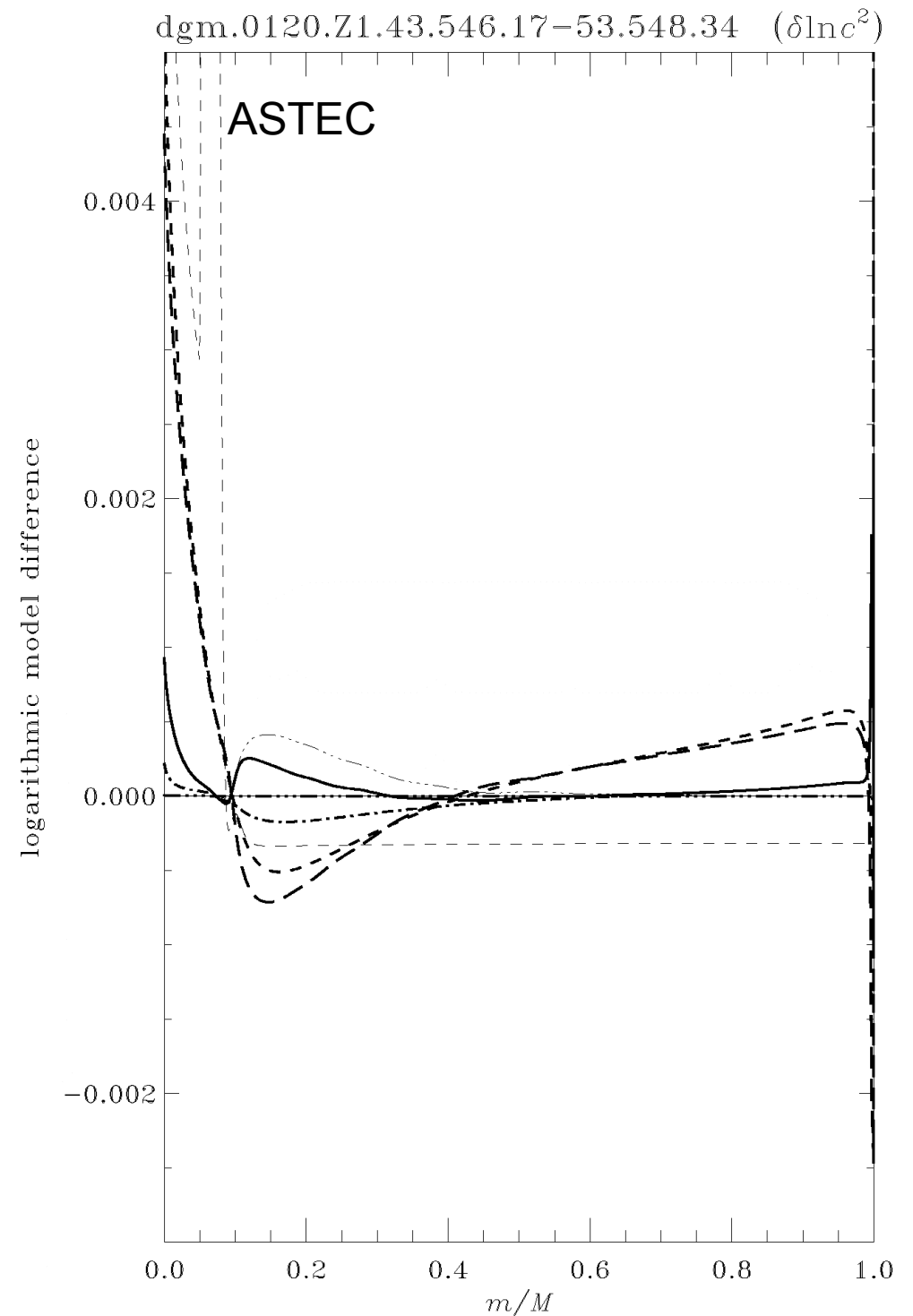
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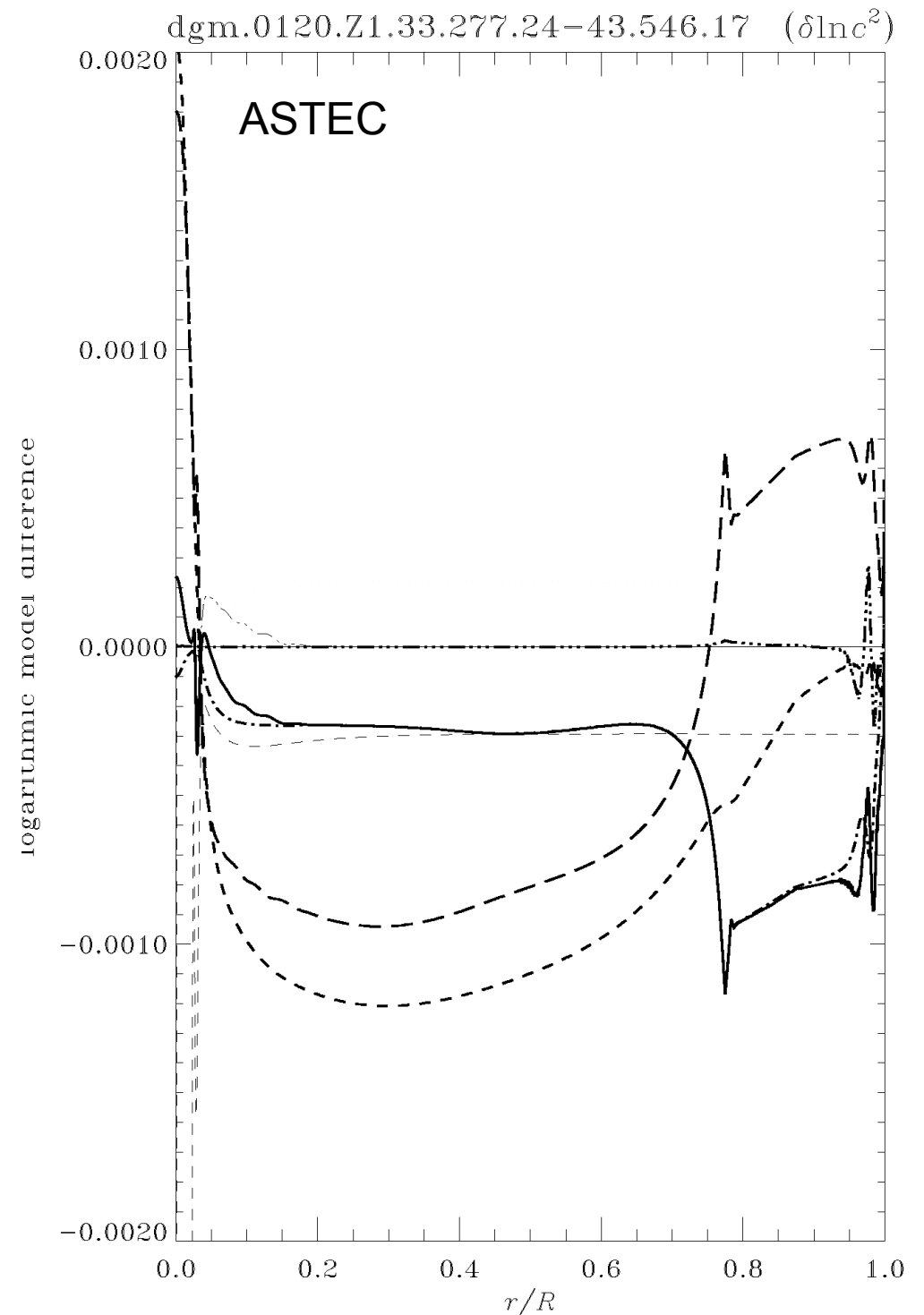
Test effect of no. timesteps:

(N_t = 277) – (N_t = 546)

(Δ y_{max} = 0.05) – (Δ y_{max} = 0.025)

Line styles:

..... : δlnT	———— : δlnq
----- : δlnp	----- : δlnL
----- : δlnρ : δX
———— : δlnc ²	
..... : δlnΓ ₁	



Case 1.5

2.0 M_{\odot} , $X_c = 0.01$,

Overshoot 0.15 H_p

^3He in equilibrium

Test effect of no. of meshpoints:

($N = 600$) – ($N = 1200$)

Line styles:

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----- : $\delta \ln p$

----- : $\delta \ln \rho$

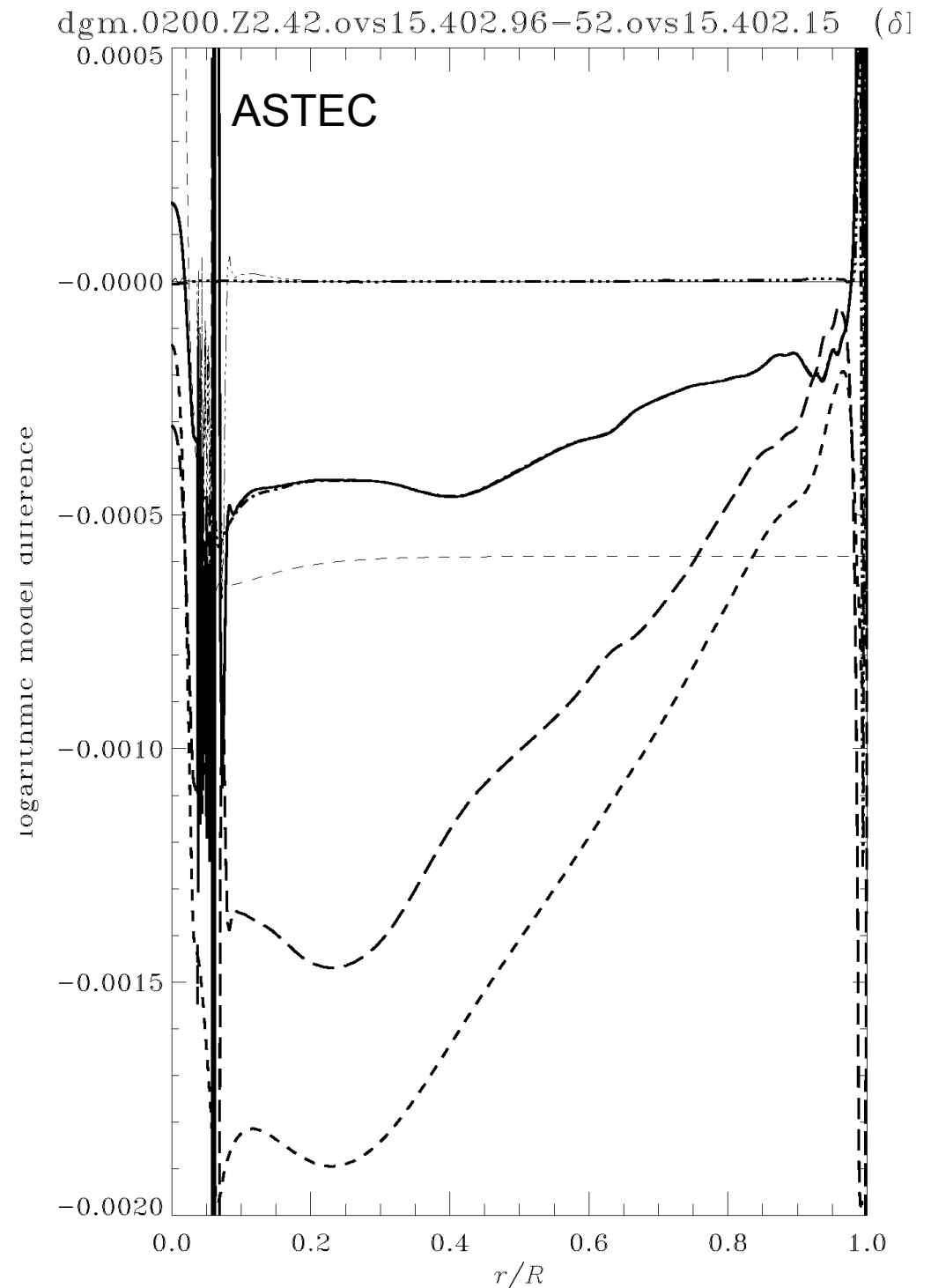
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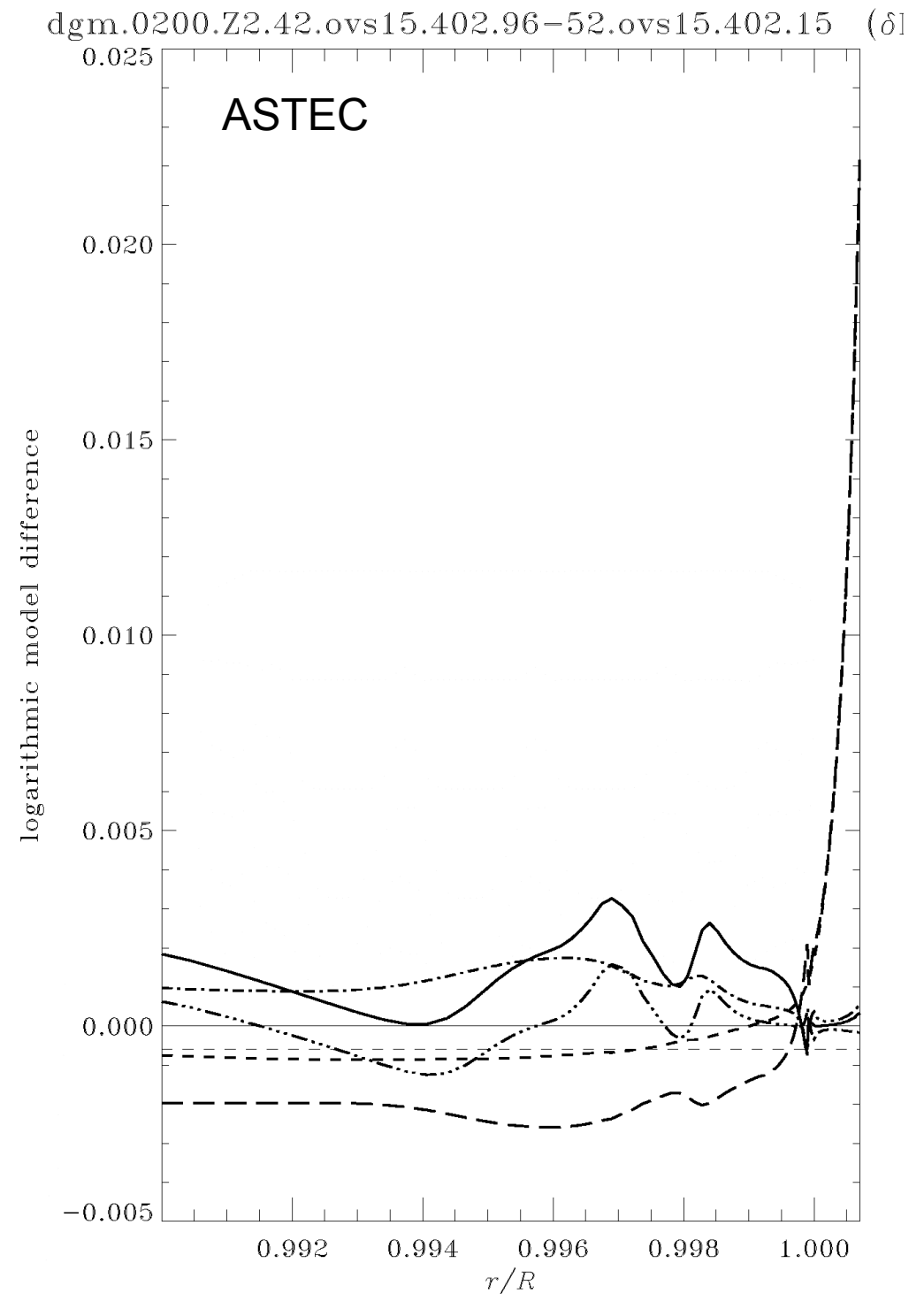
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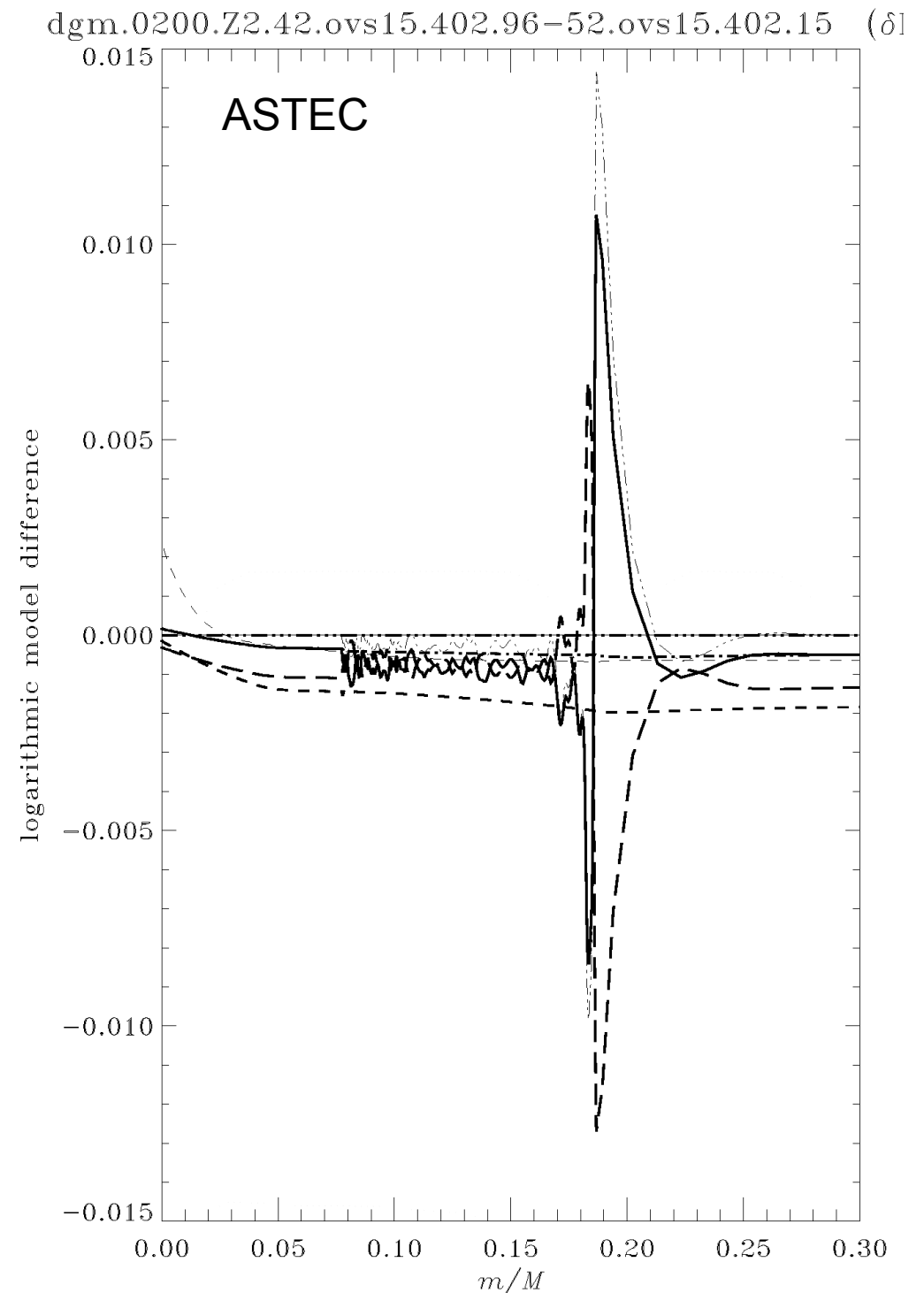
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Case 1.5

2.0 M_⊙, $X_c = 0.01$,

Overshoot 0.15 H_p

³He in equilibrium

Test effect of no. of timesteps:

($N_t = 208$) – ($N_t = 402$)

Line styles:

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----- : $\delta \ln p$

----- : $\delta \ln \rho$

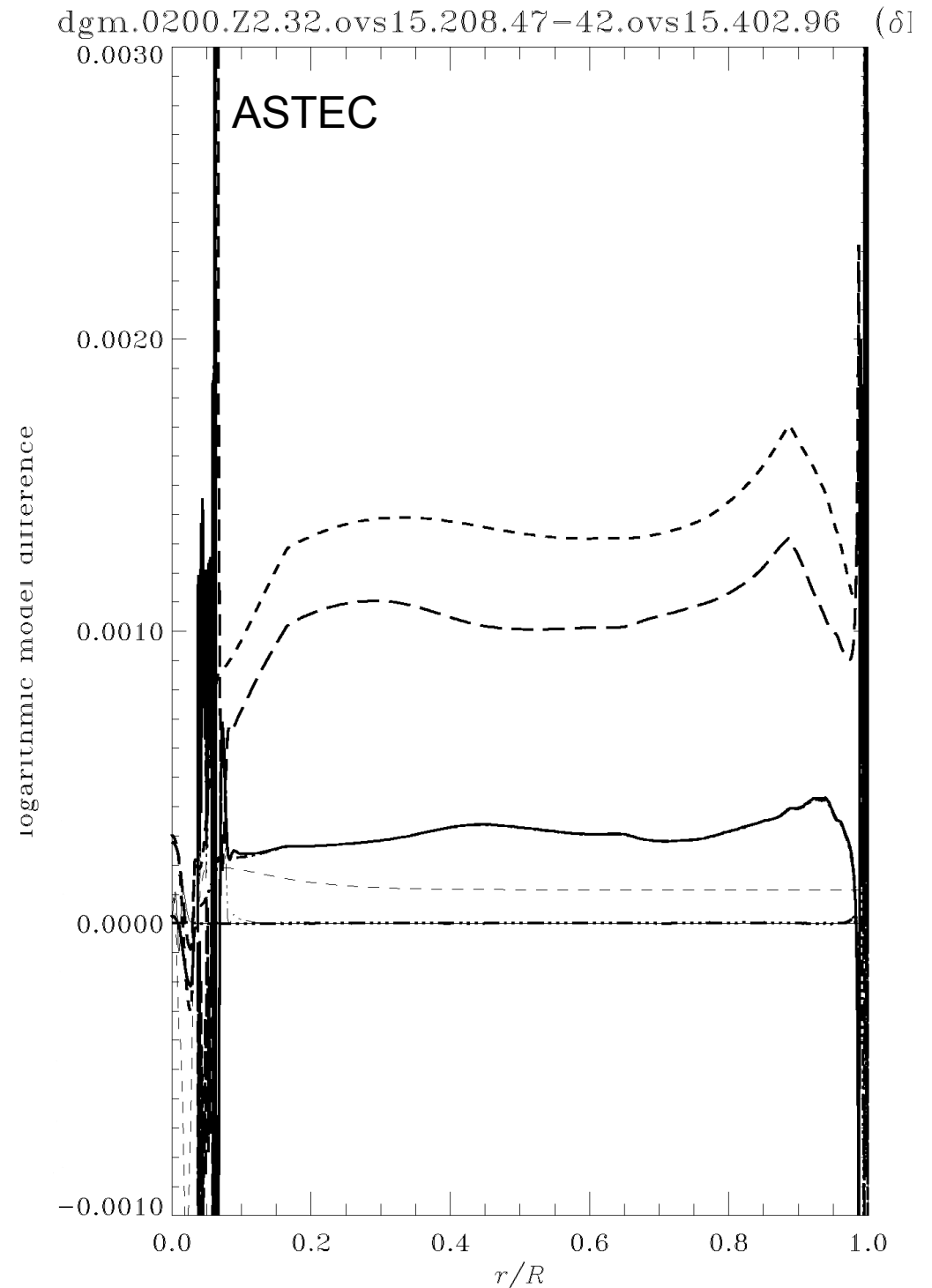
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Case 1.5

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Overshoot 0.15 H_p

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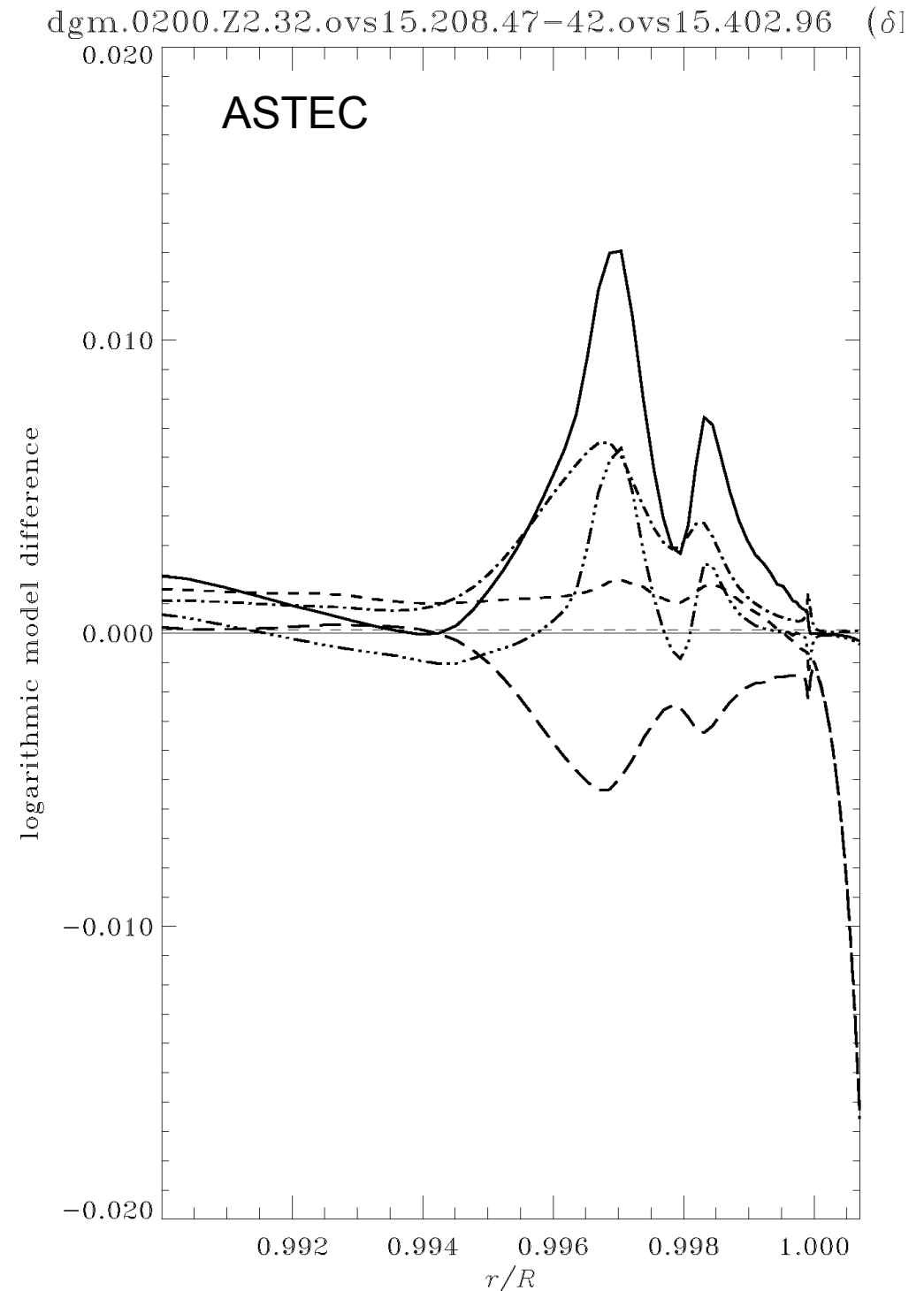
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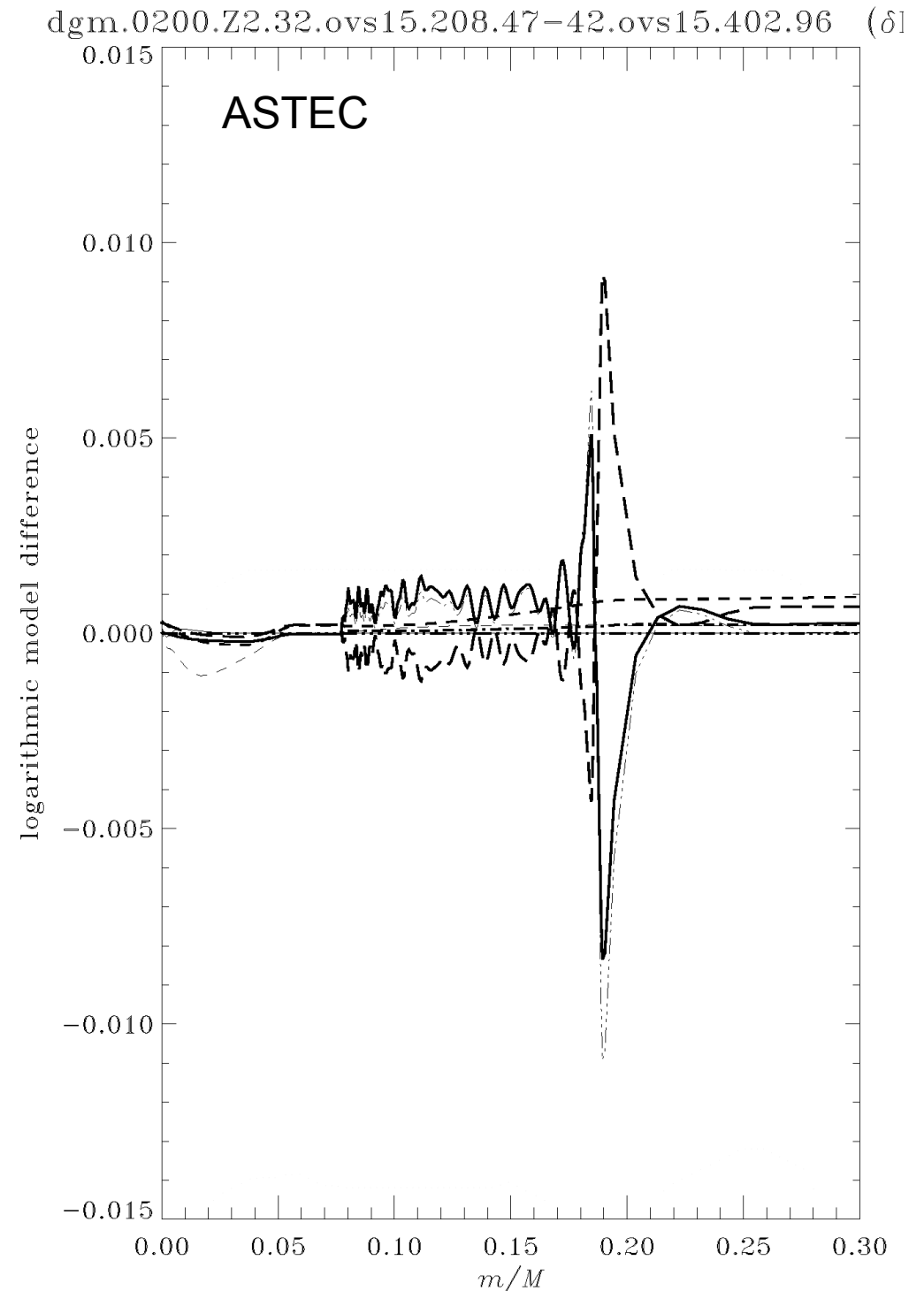
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----- : δX



Case 1.1

$0.9 M_{\odot}$, $X_c = 0.35$,

TGEC

Test effect of no. of timesteps:
($\Delta t = 2000$ yr) – ($\Delta t = 1800$ yr)

Continuous: $\delta \ln T$

Dotted: $\delta \ln p$

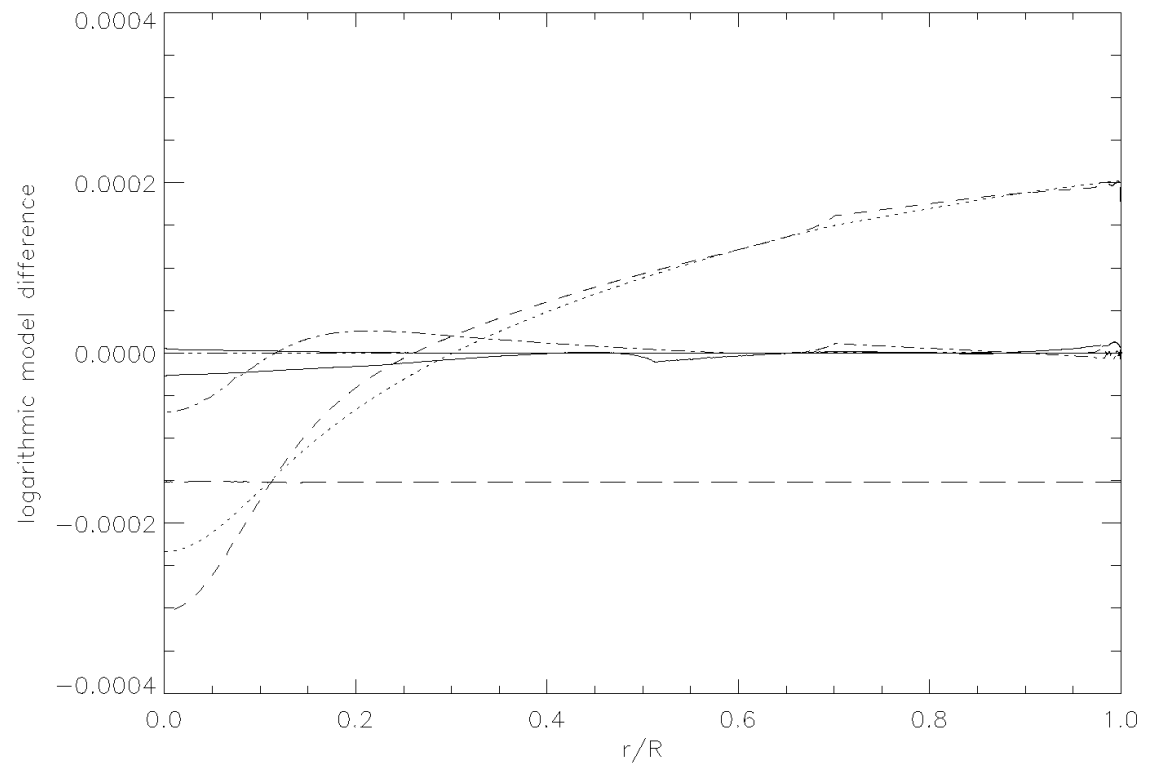
Dashed: $\delta \ln \rho$

Dot-dashed: $\delta \ln c^2$

3dot-dashed: $\delta \ln \Gamma_1$

Long-dashed: $\delta \ln L$

Thick continuous: $\delta \ln X$



Case 1.1

$0.9 M_{\odot}$, $X_c = 0.35$,

TGEC

Test effect of no. of timesteps:
($\Delta t = 2000$ yr) – ($\Delta t = 2200$ yr)

Continuous: $\delta \ln T$

Dotted: $\delta \ln p$

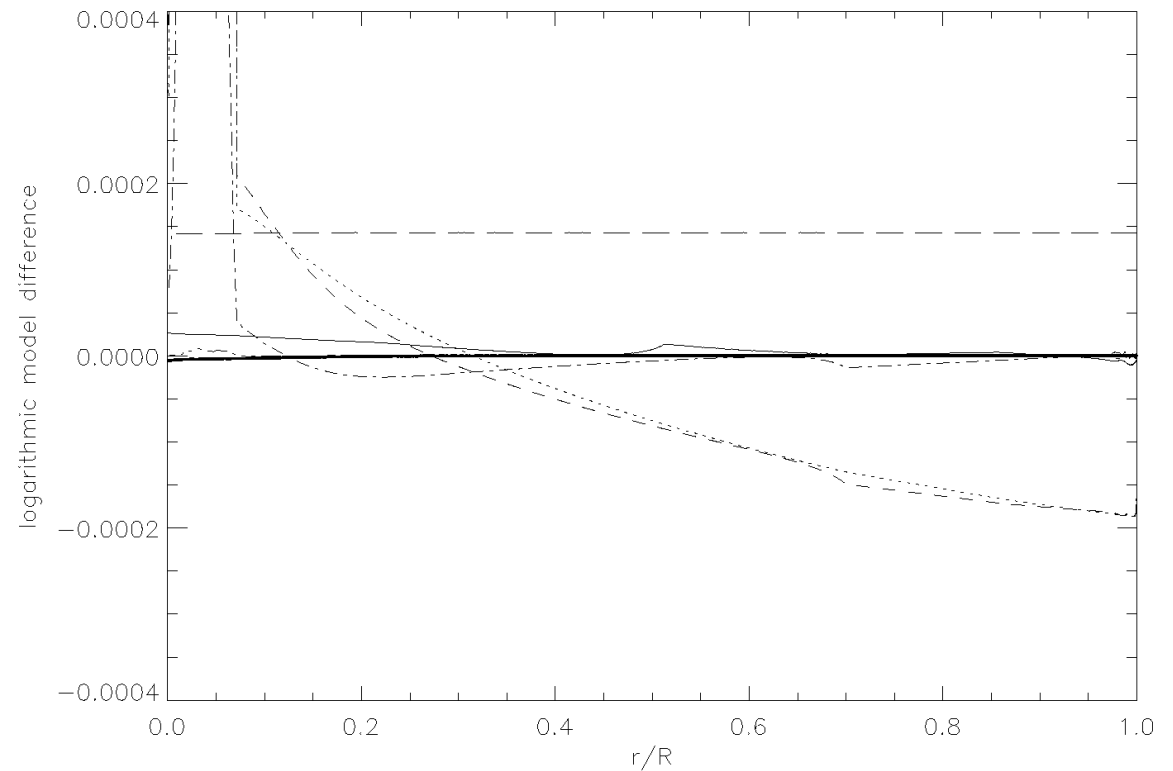
Dashed: $\delta \ln \rho$

Dot-dashed: $\delta \ln c^2$

3dot-dashed: $\delta \ln \Gamma_1$

Long-dashed: $\delta \ln L$

Thick continuous: $\delta \ln X$



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$0.9 M_{\odot}$, $X_c = 0.35$,

TGEC

Test effect of no. of timesteps:

($\Delta t = 2000$ yr) – ($\Delta t = 2200$ yr)

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Dotted: $\delta \ln p$

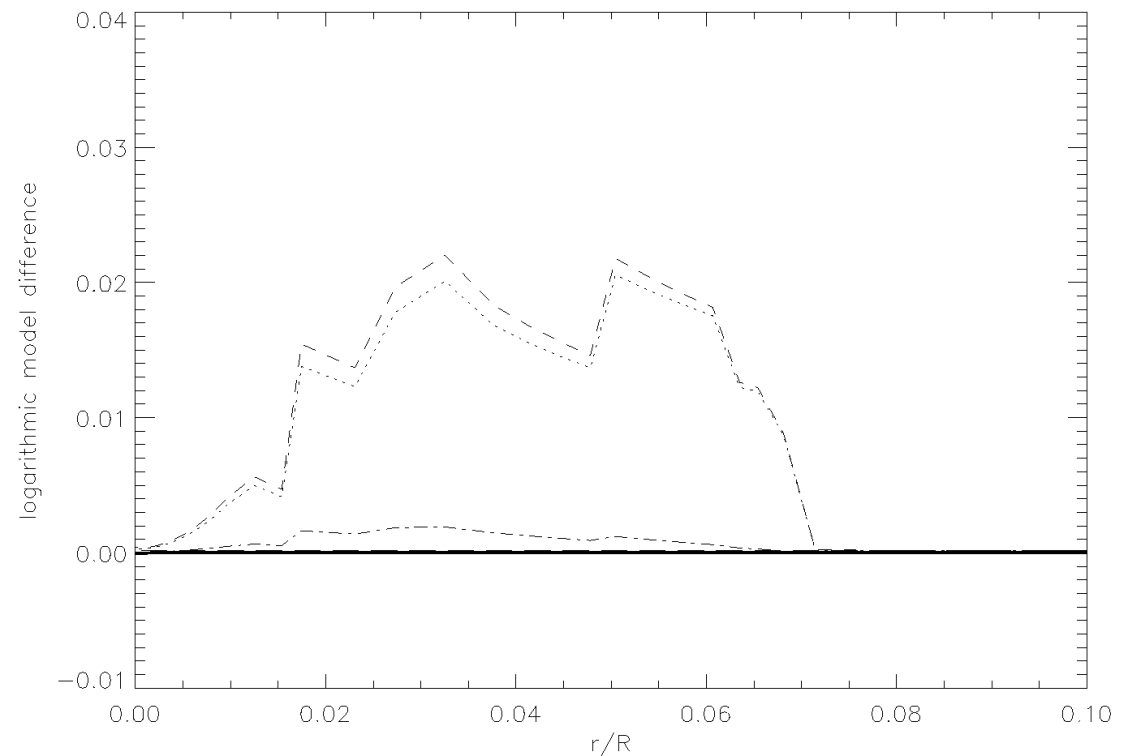
Dashed: $\delta \ln \rho$

Dot-dashed: $\delta \ln c^2$

3dot-dashed: $\delta \ln \Gamma_1$

Long-dashed: $\delta \ln L$

Thick continuous: $\delta \ln X$



Physics comparisons

Evaluate physics (EOS, opacity, energy-generation rate, rate of composition change, ..., at fixed T , ρ , X_i

Examples: comparing CESAM and CLES with ASTEC, showing, e.g.,

$$\ln(\kappa_{\text{ASTEC}}(\rho_{\text{CESAM}}, T_{\text{CESAM}}, \dots) / \kappa_{\text{CESAM}})$$

CESAM, Case 1.1

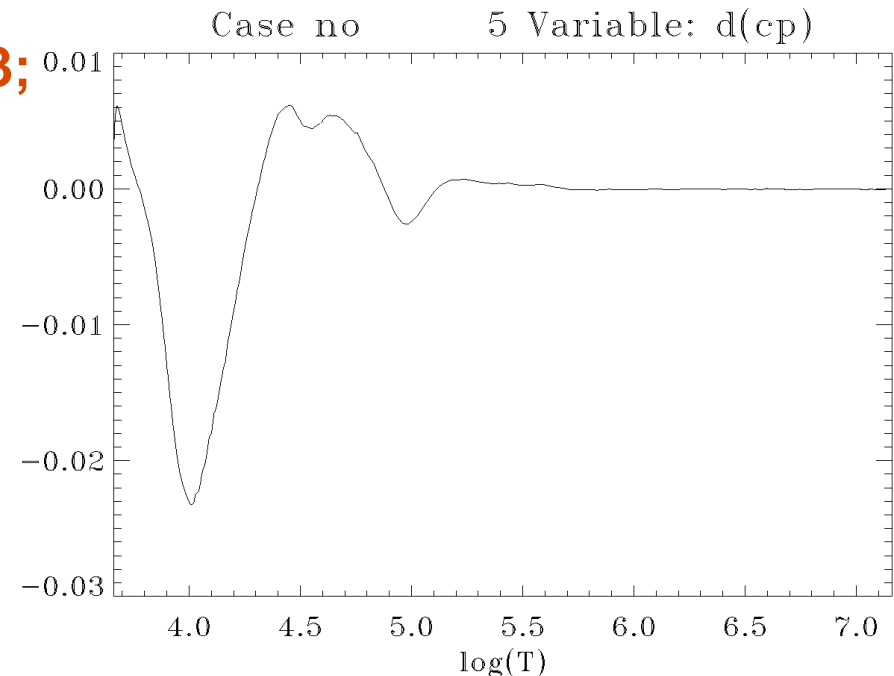
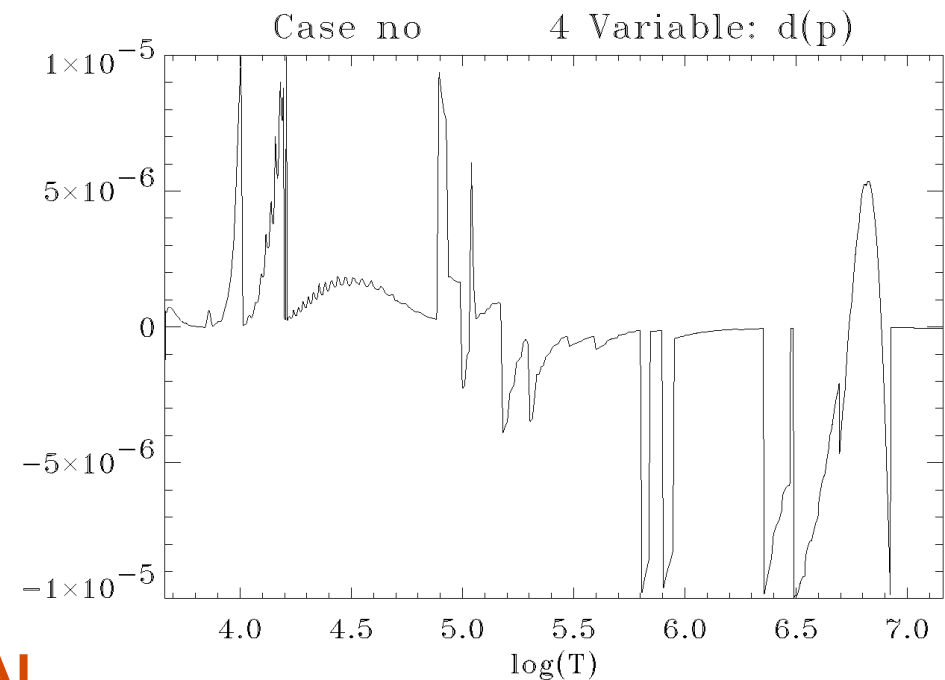
Note: consistency problems in OPAL.

**See also Boothroyd & Sackman (2003;
ApJ 583, 1004)**

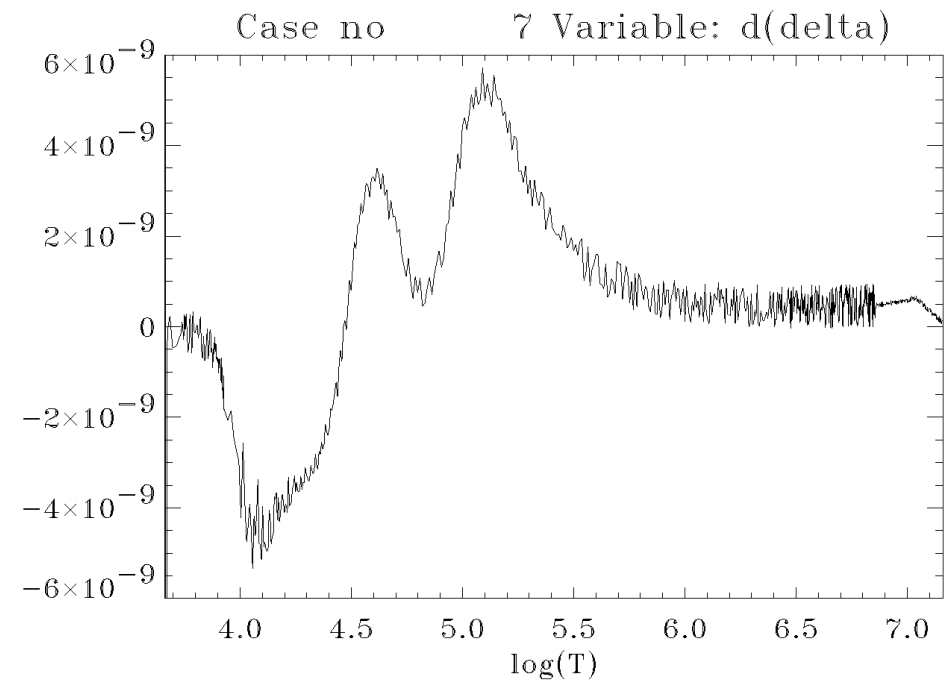
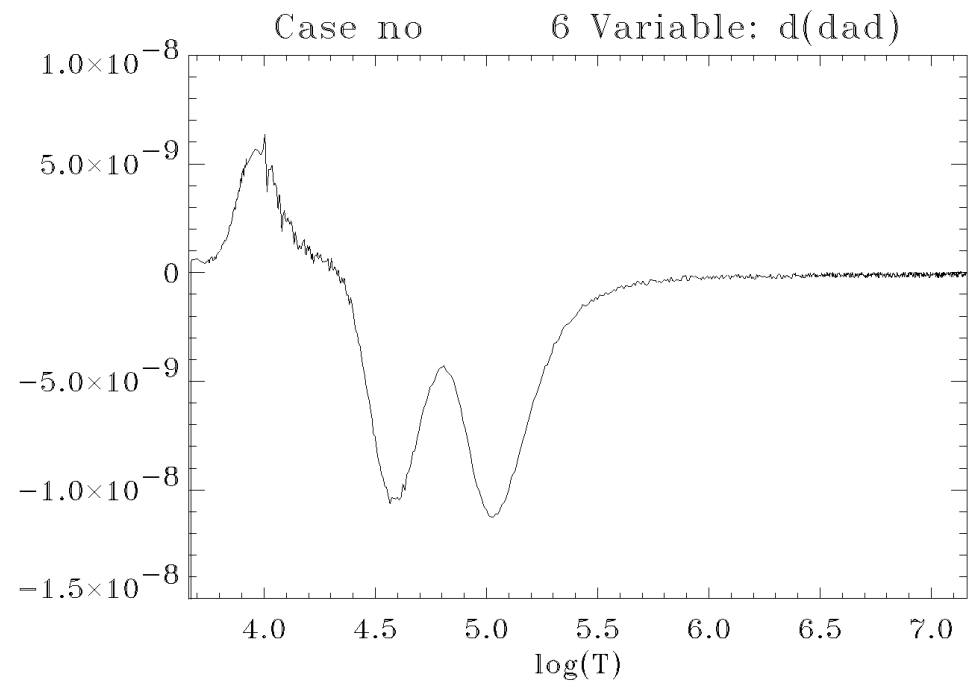
In ASTEC implementation:

Directly from OPAL: p , r_{ad} , δ , α , Γ_1

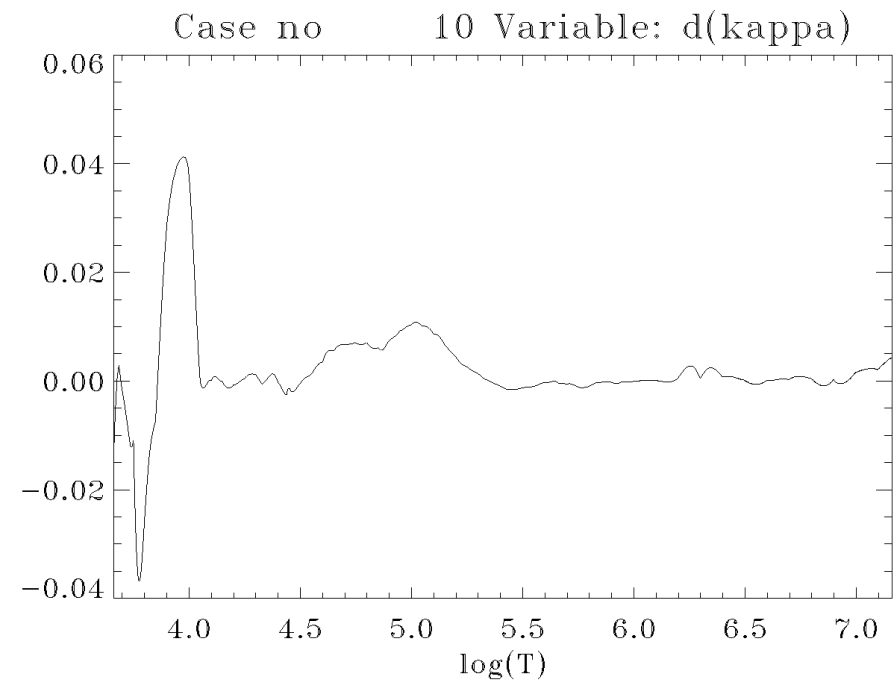
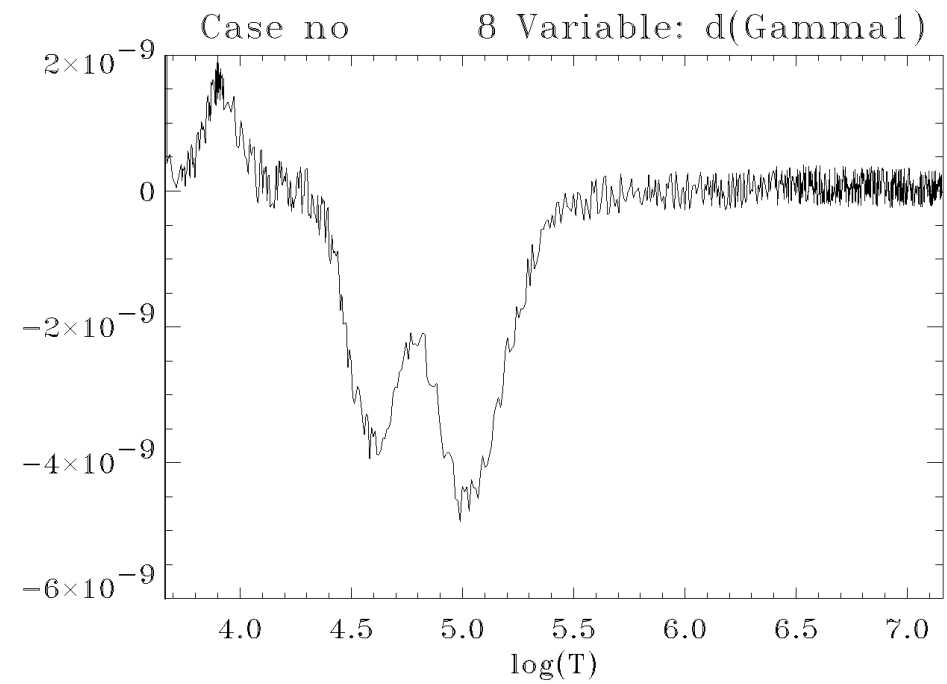
$$c_p = c_v + p \delta^2 / (\rho T \alpha)$$



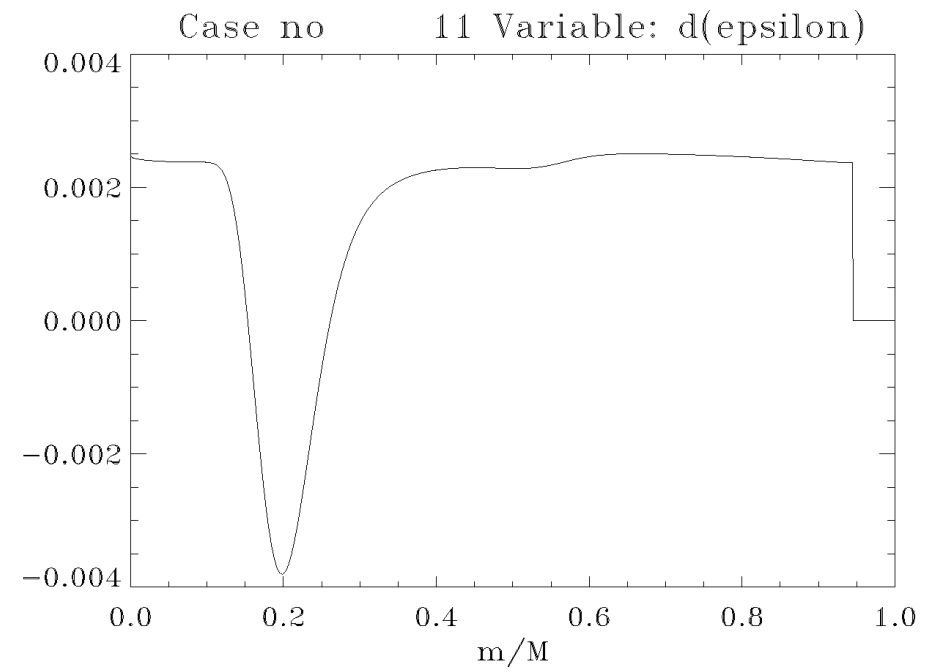
CESAM, Case 1.1



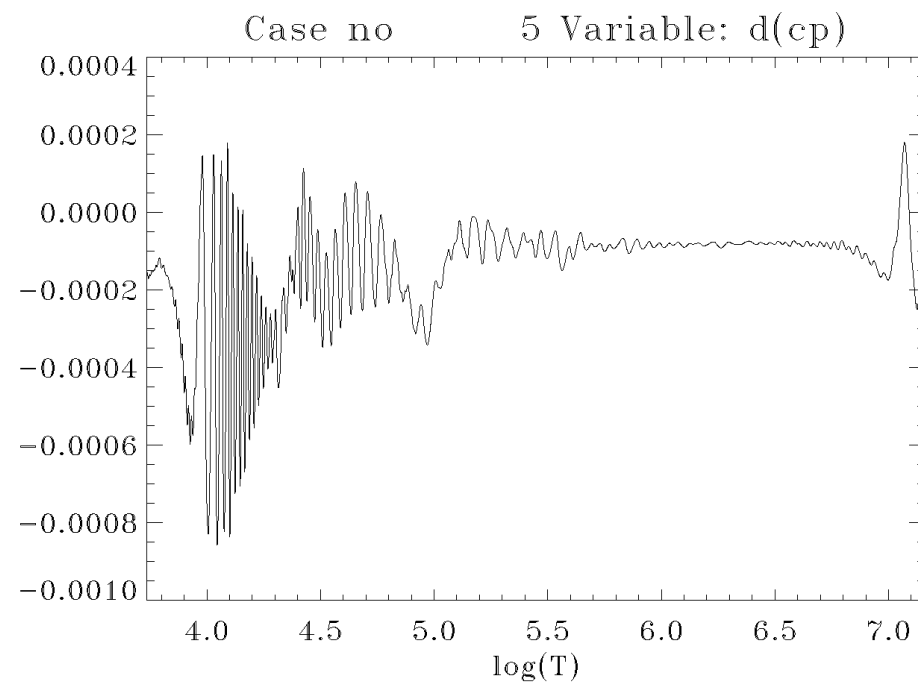
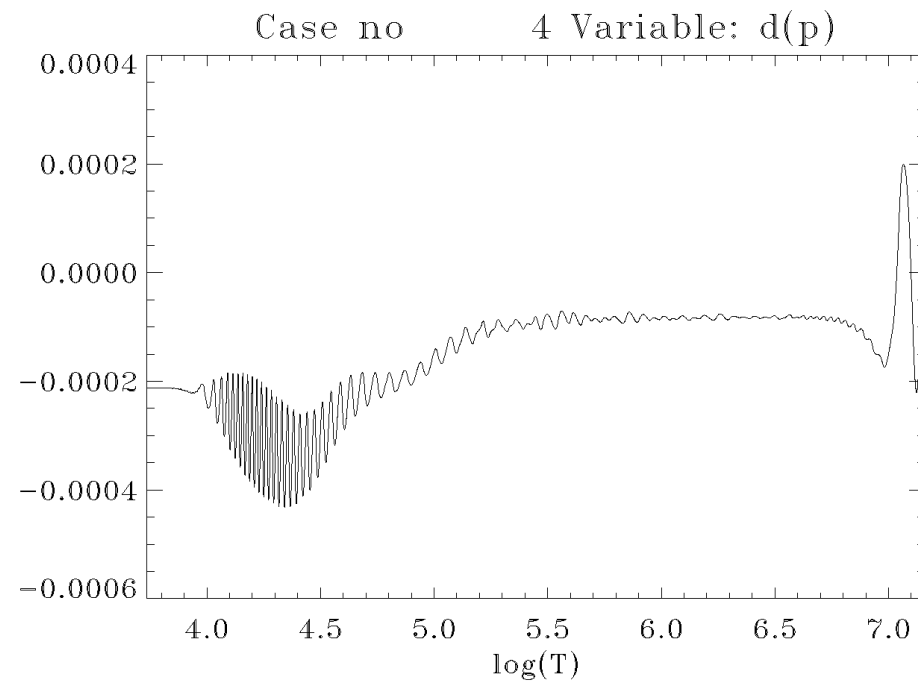
CESAM, Case 1.1



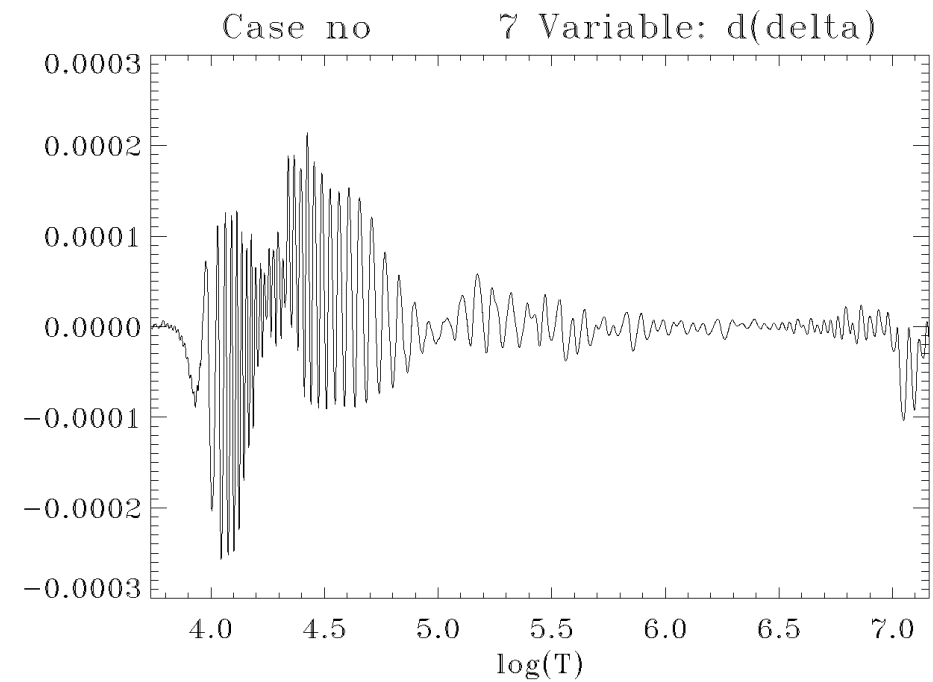
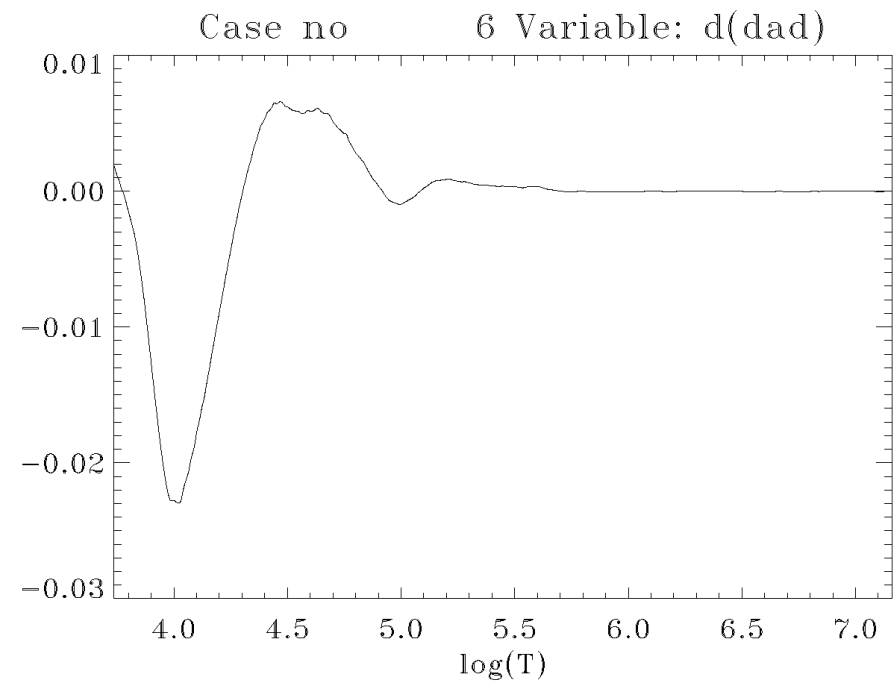
CESAM, Case 1.1



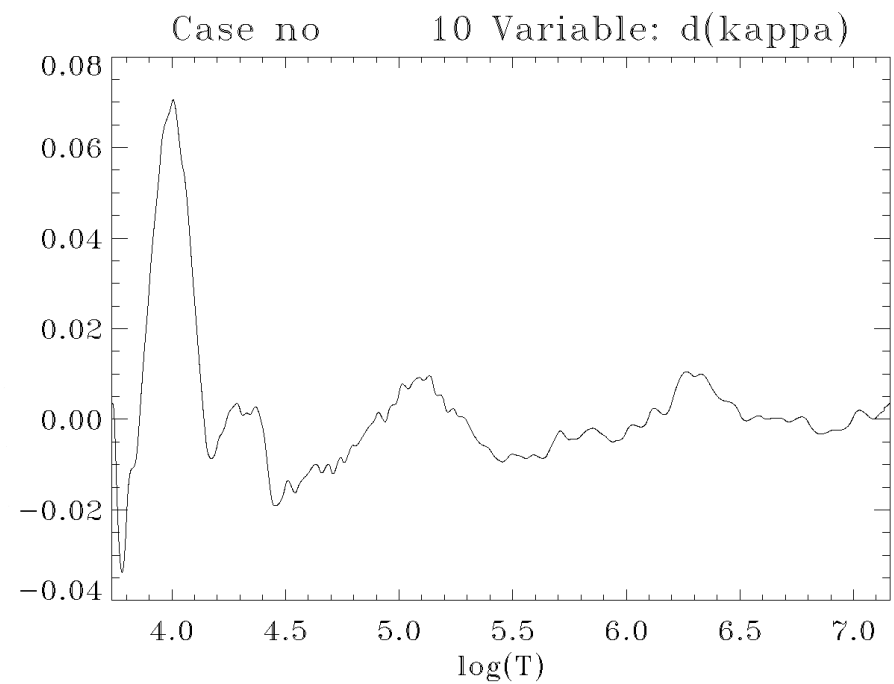
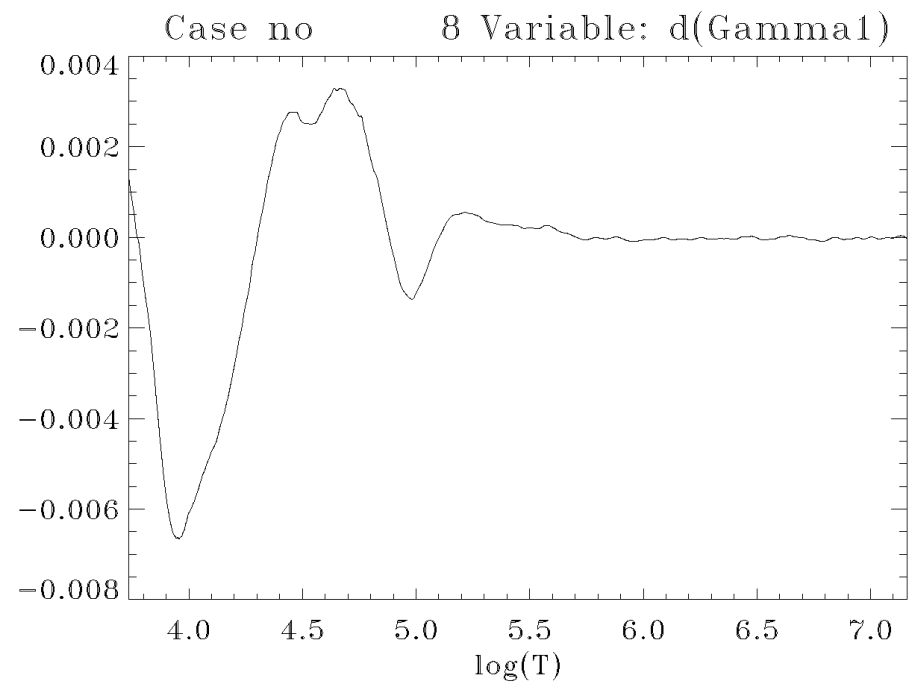
CLES, Case 1.1



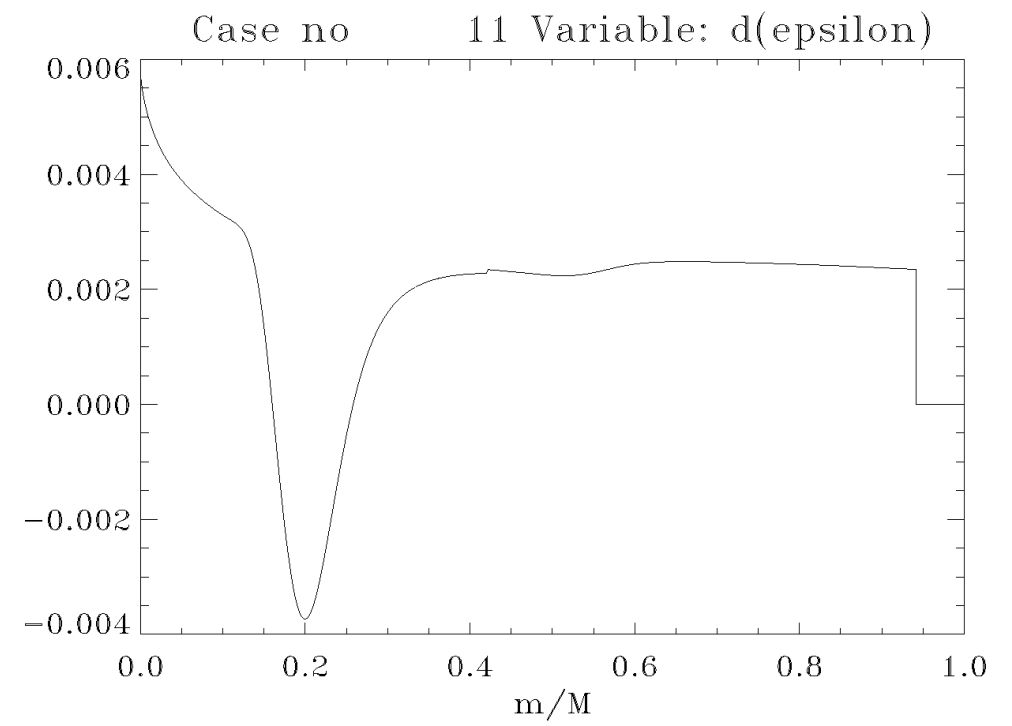
CLES, Case 1.1



CLES, Case 1.1



CLES, Case 1.1



Main project: compare different codes

- Evolution tracks
- Global parameters for selected models
- Detailed comparison of structure
- Comparison of oscillation frequencies

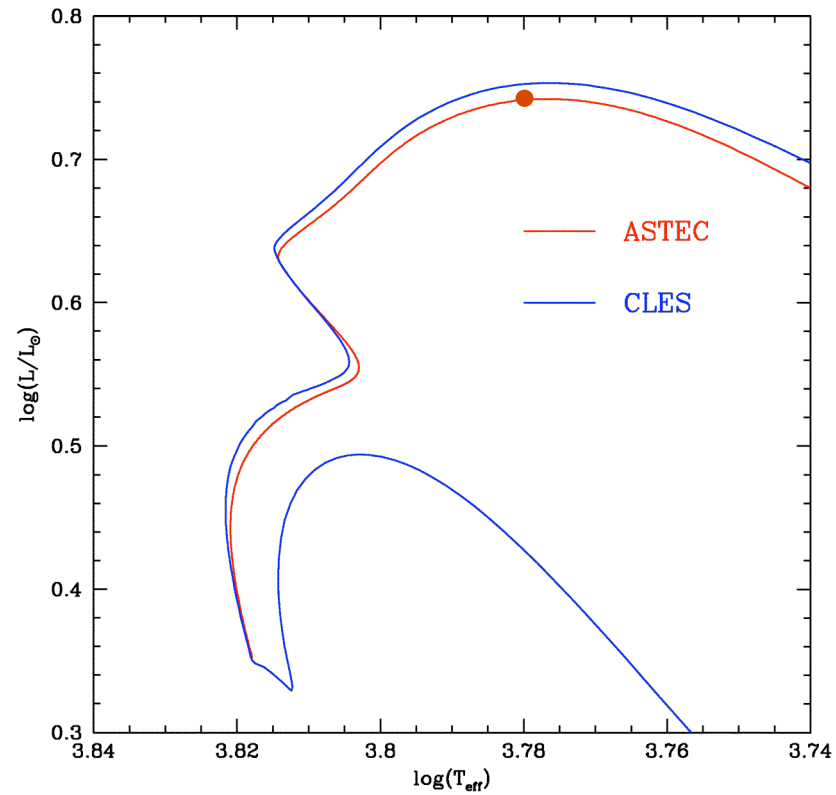
CLES and ASTEC

Case 1.3

1.2 M \odot

$X_0 = 0.73$, $Z_0 = 0.01$

$M_{\text{HeC}} = 0.1 M_{\odot}$



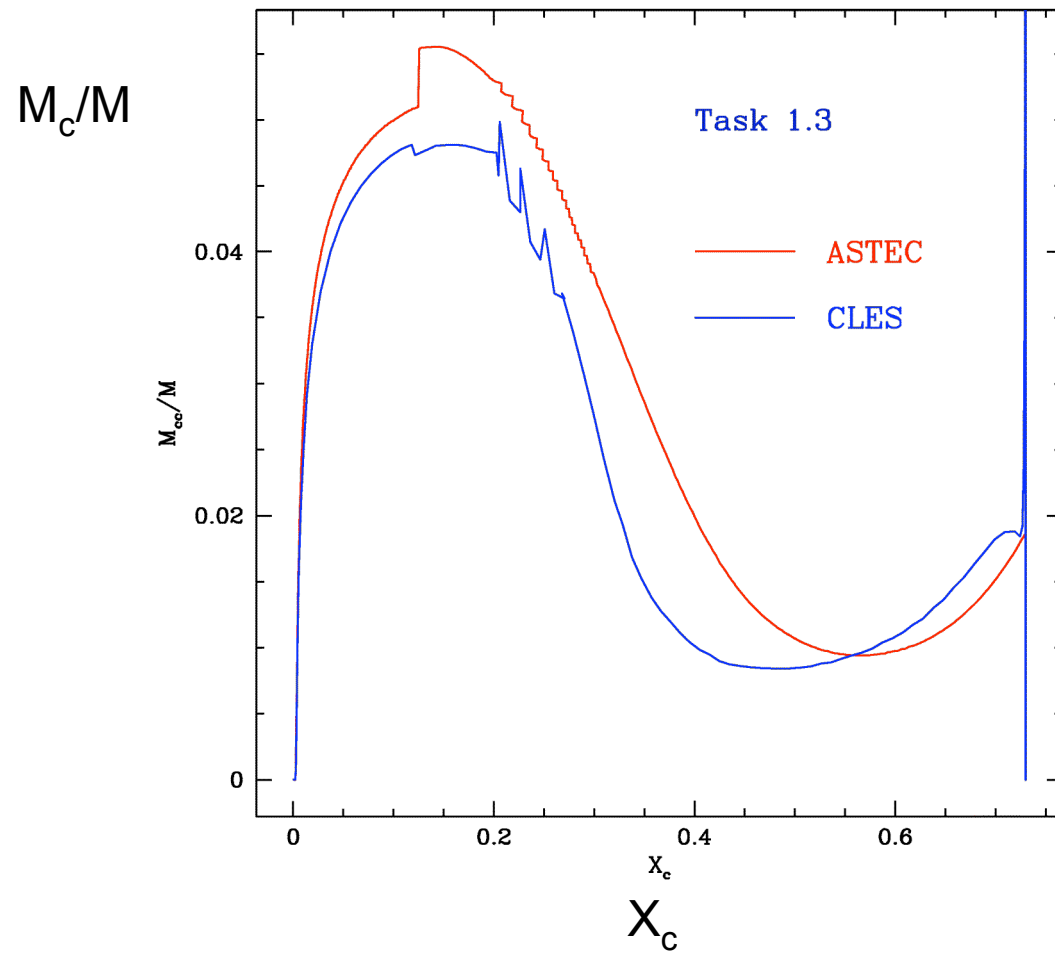
CLES and ASTEC

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CLES, CESAM and ASTEC

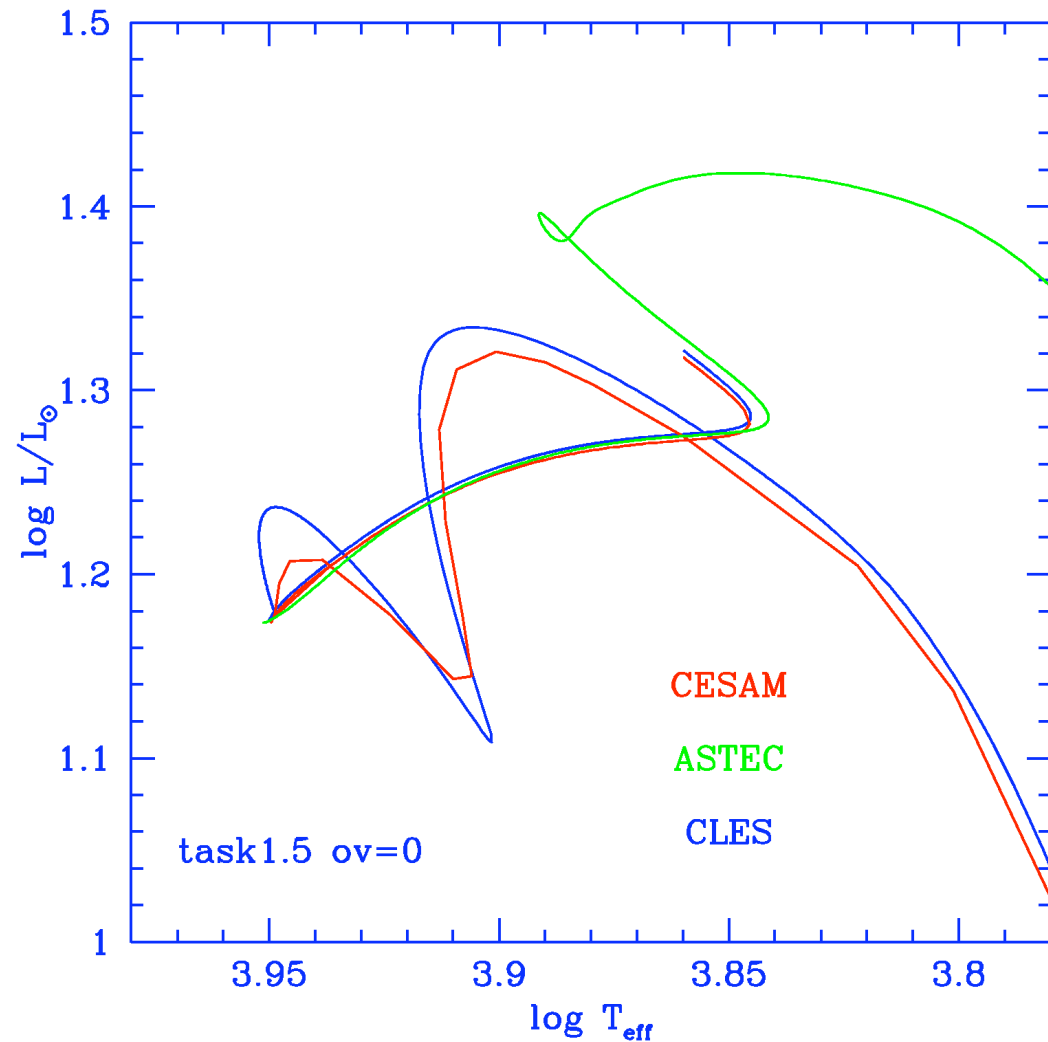
Case 1.5n

2.0 M-

$X_0 = 0.72$, $Z_0 = 0.02$

$X_c = 0.01$

No overshoot



CLES, CESAM and ASTEC

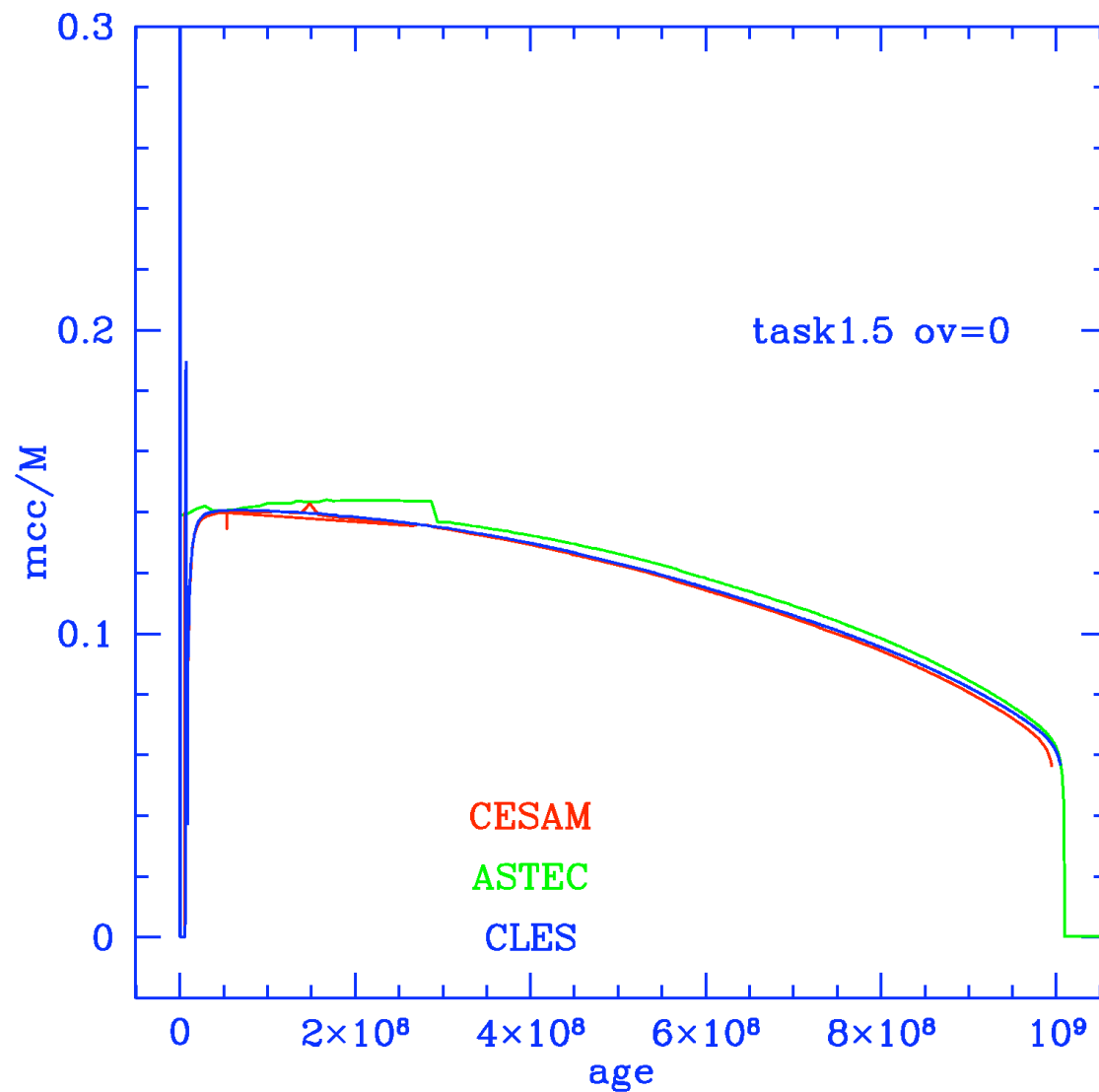
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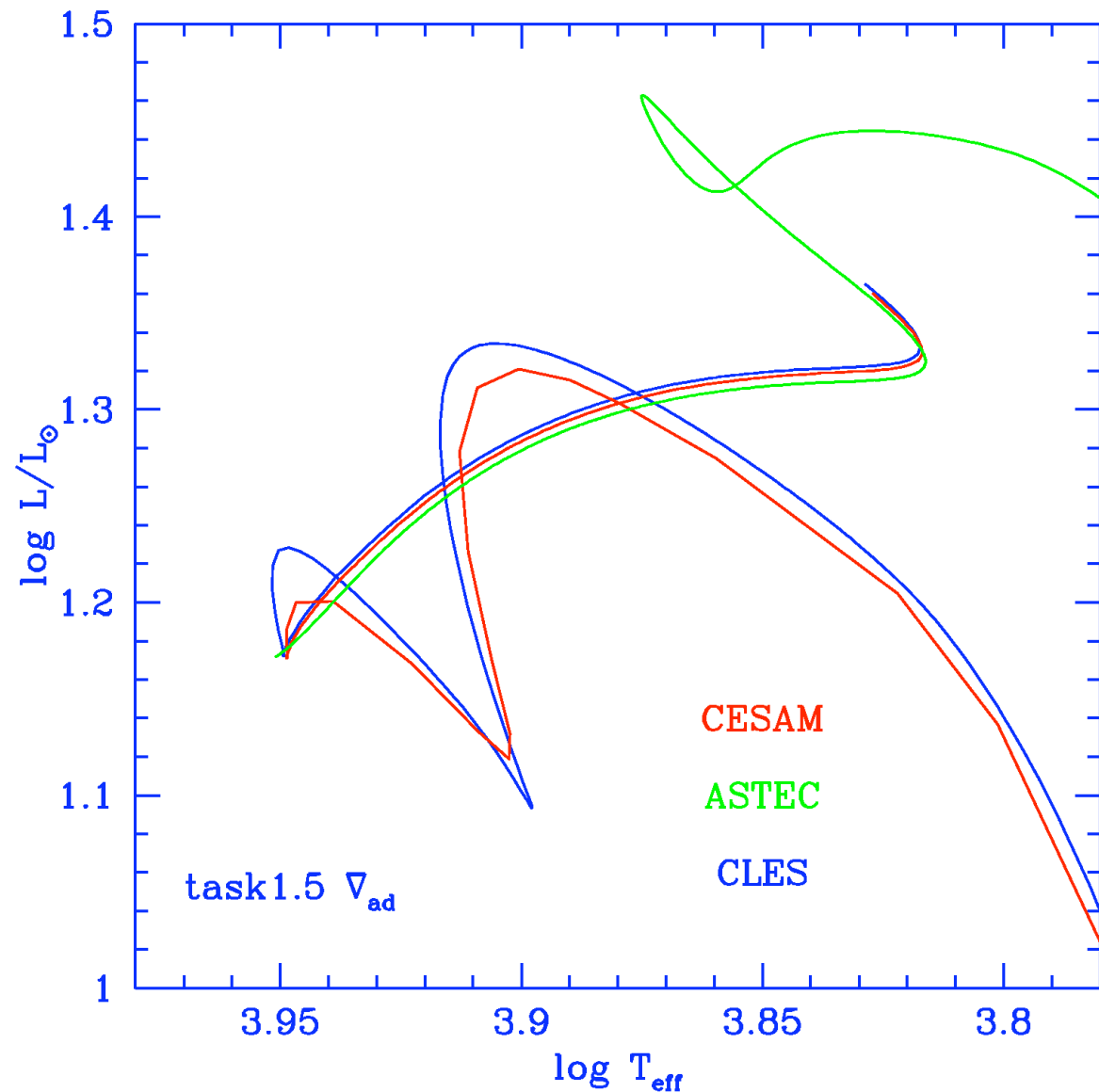
Case 1.5n

2.0 M-

$X_0 = 0.72$, $Z_0 = 0.02$

$X_c = 0.01$

Overshoot 0.15 H_p



CLES, CESAM and ASTEC

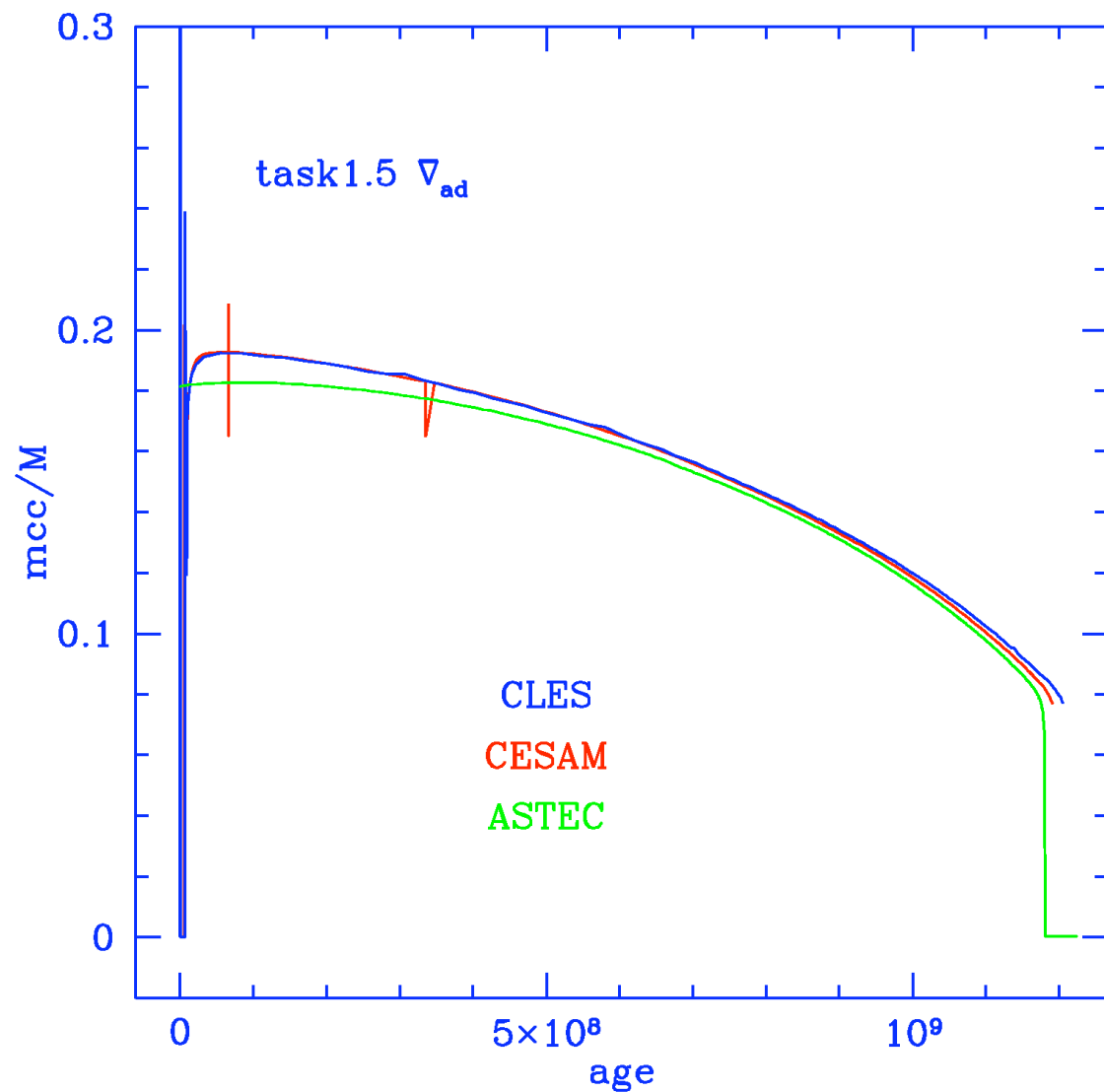
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$X_0 = 0.72$, $Z_0 = 0.02$

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Detailed model comparison

- Global quantities
- Differences at fixed m/M , plotted against m/M or r/R
- Differences at fixed r/R might be more illustrative for effects on oscillations (but not used yet)

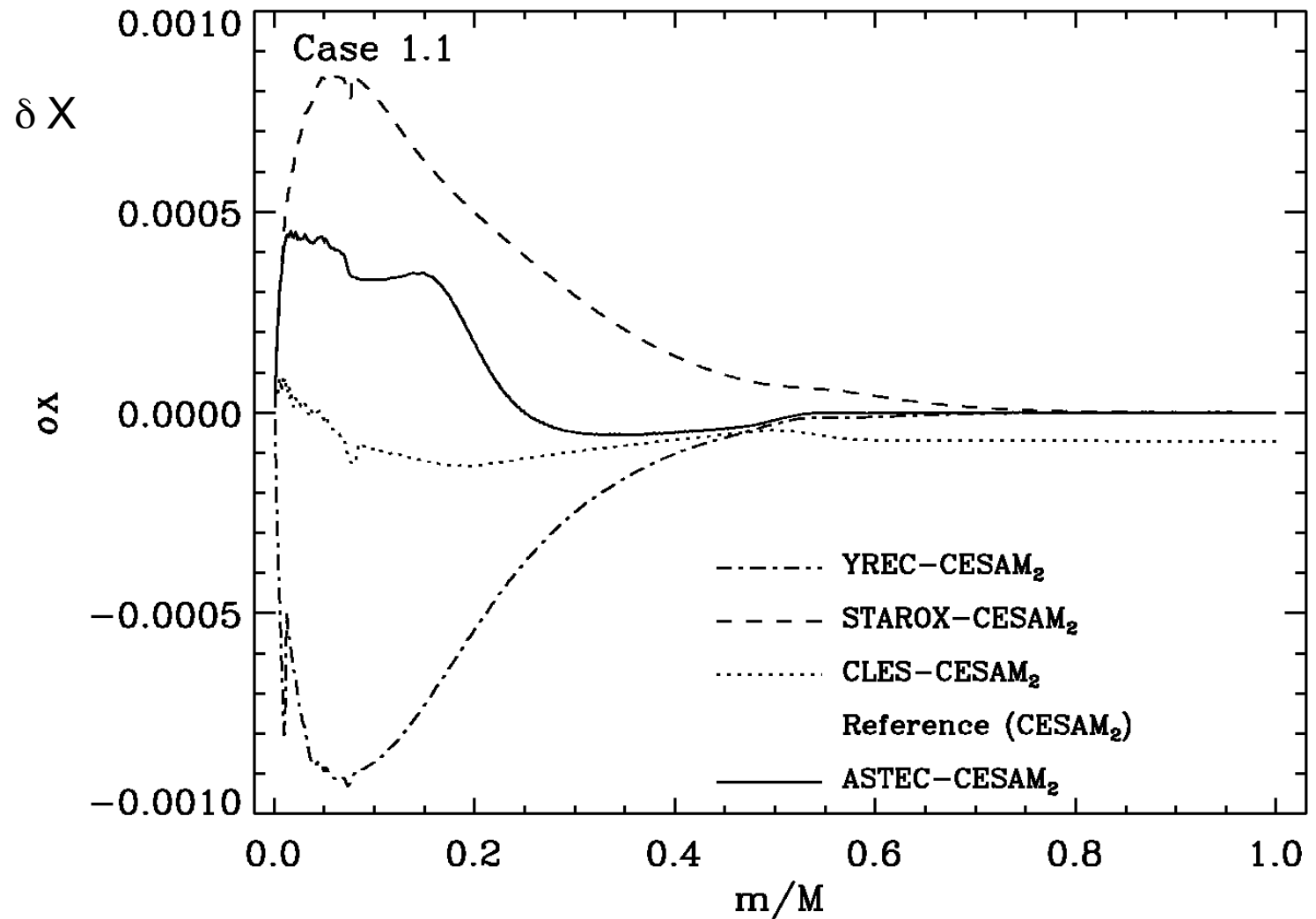
Hydrogen abundance

0.9 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.35$



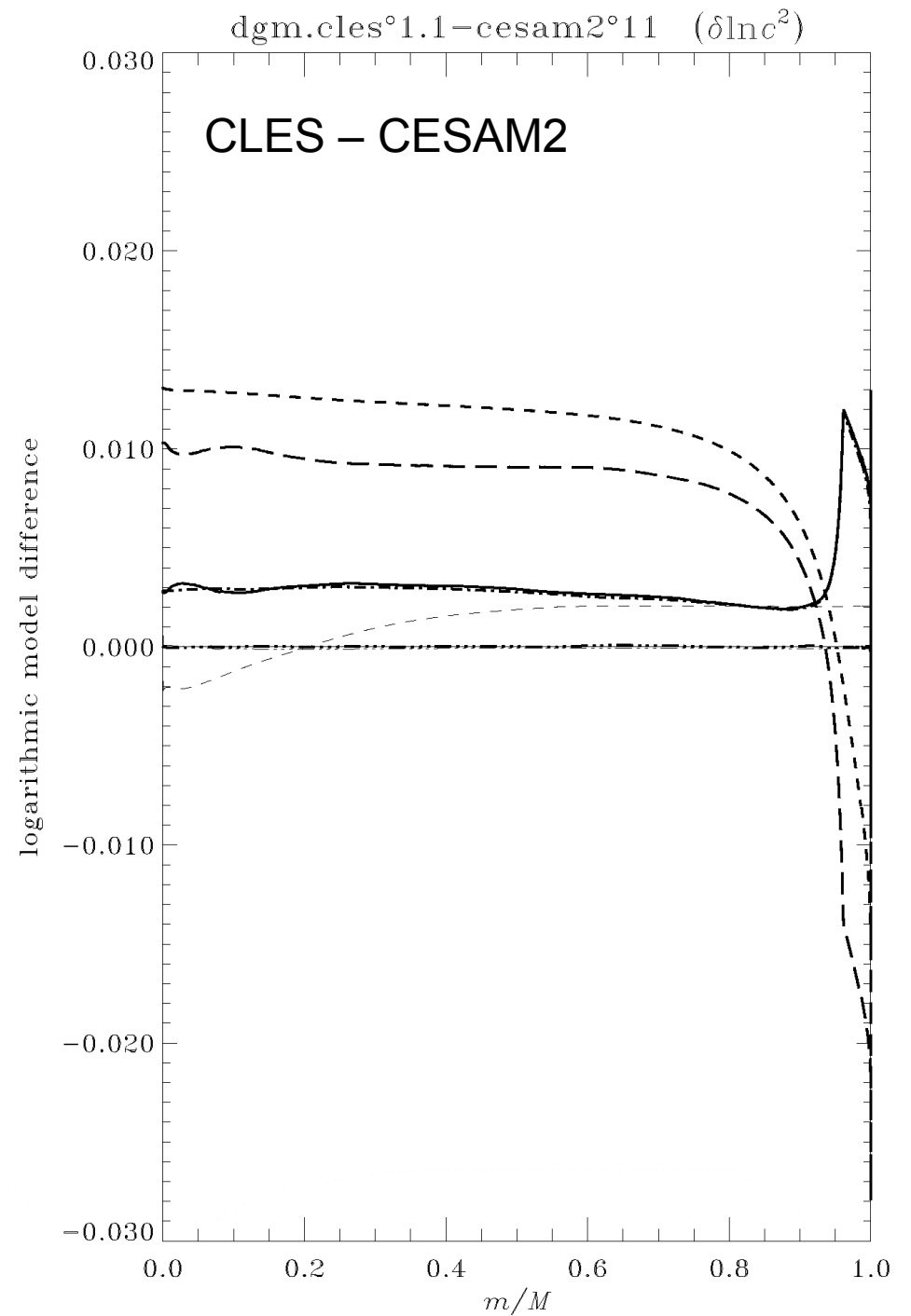
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0.9 M_{-} , $X_c = 0.35$

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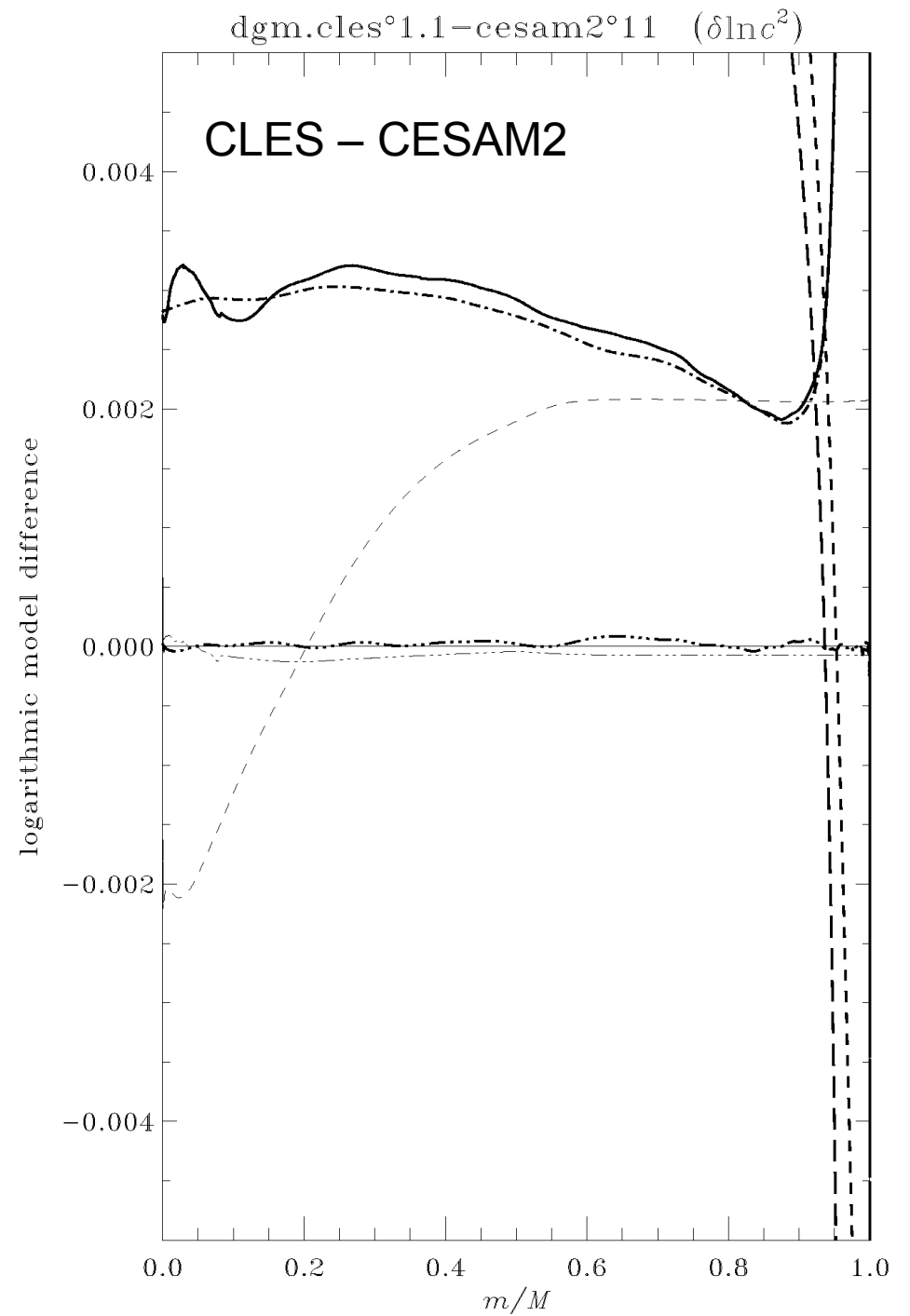
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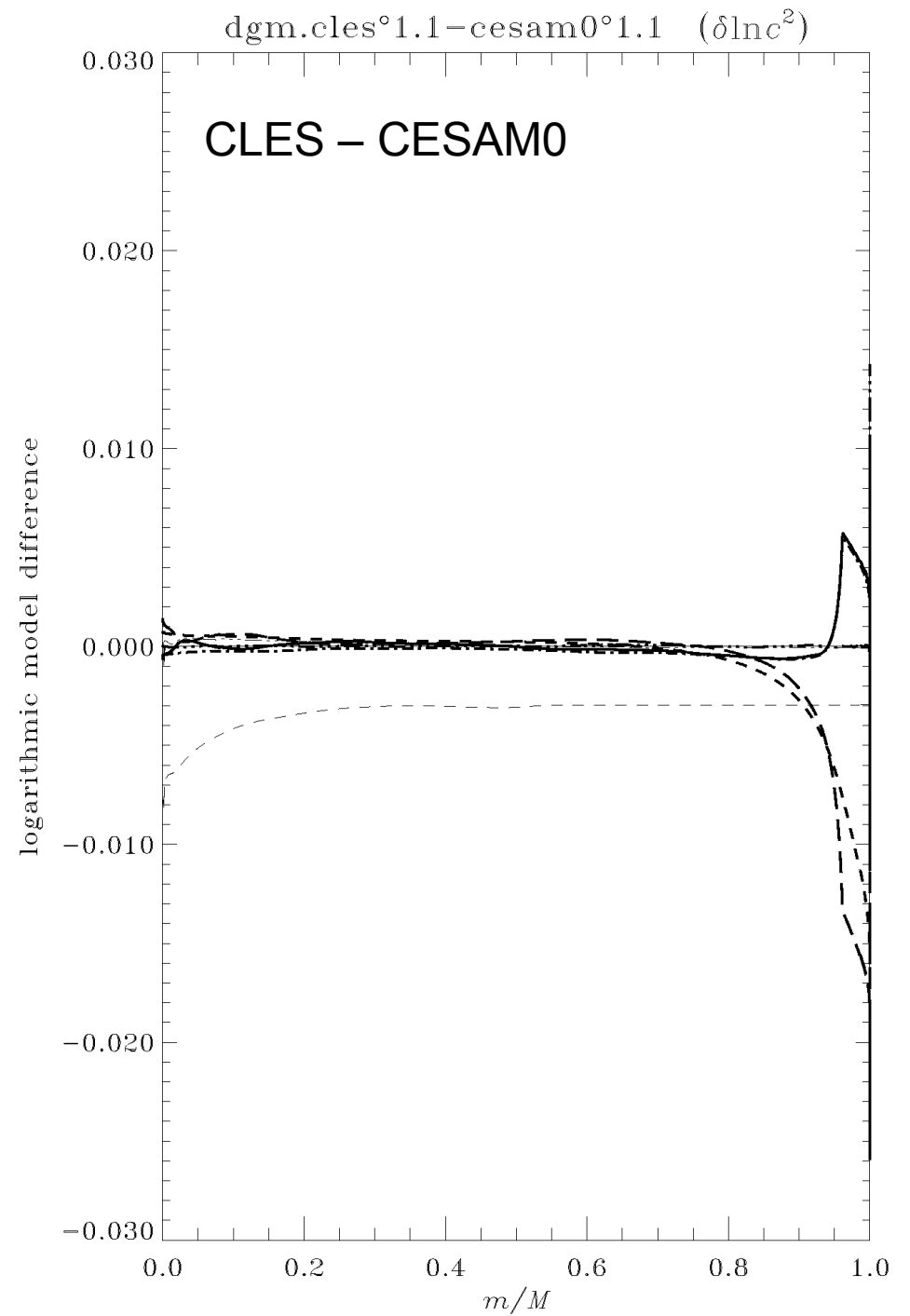
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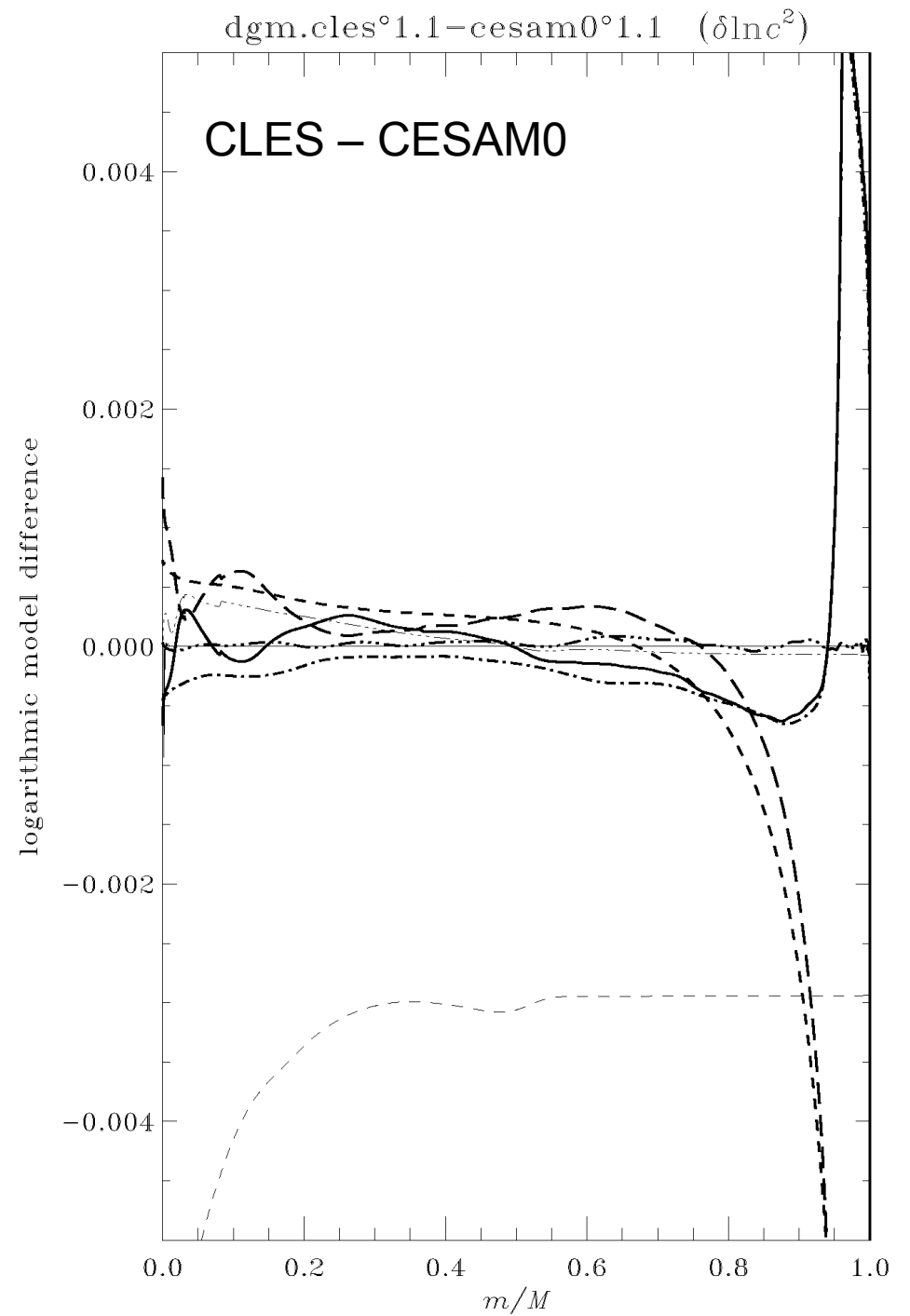
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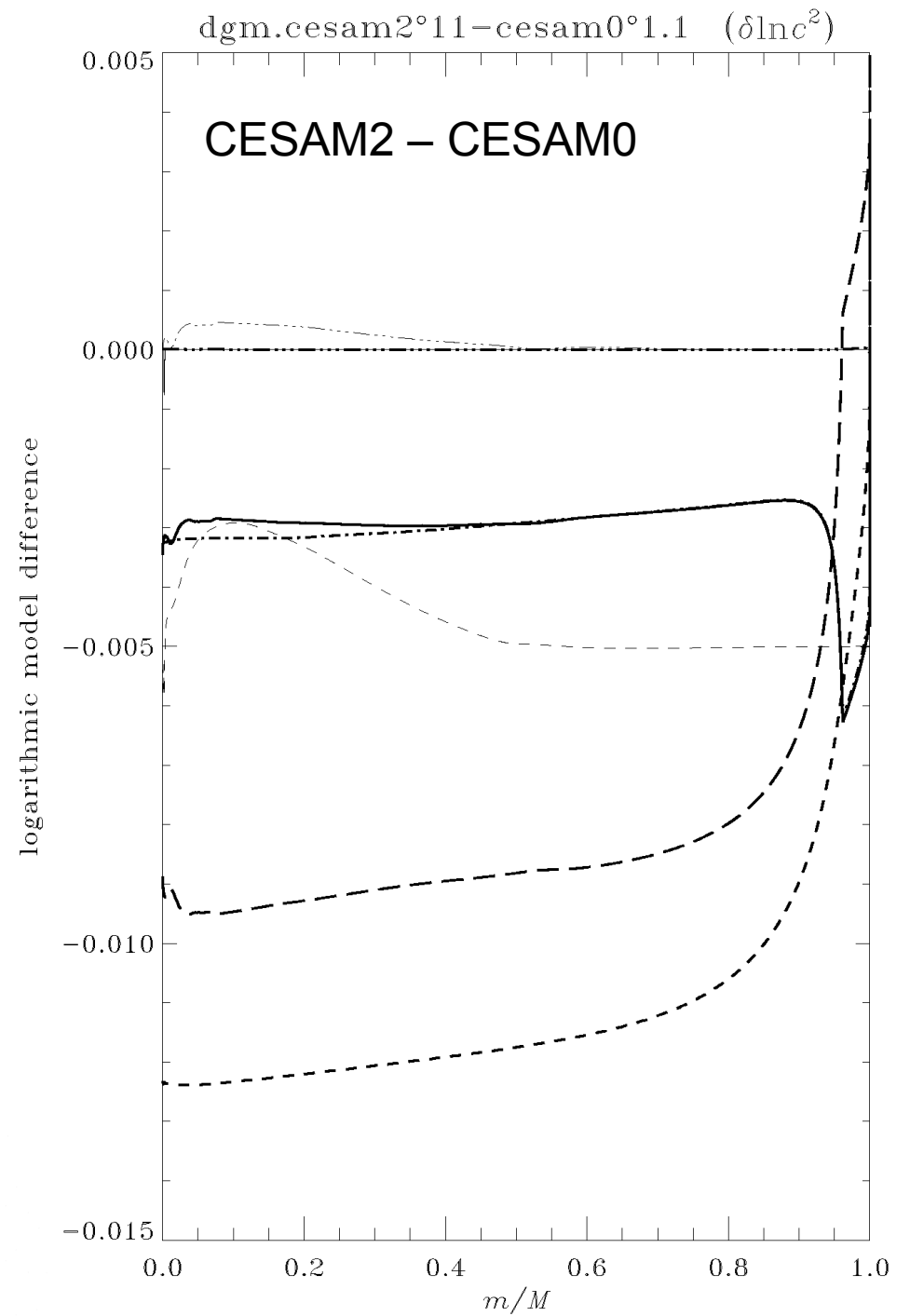
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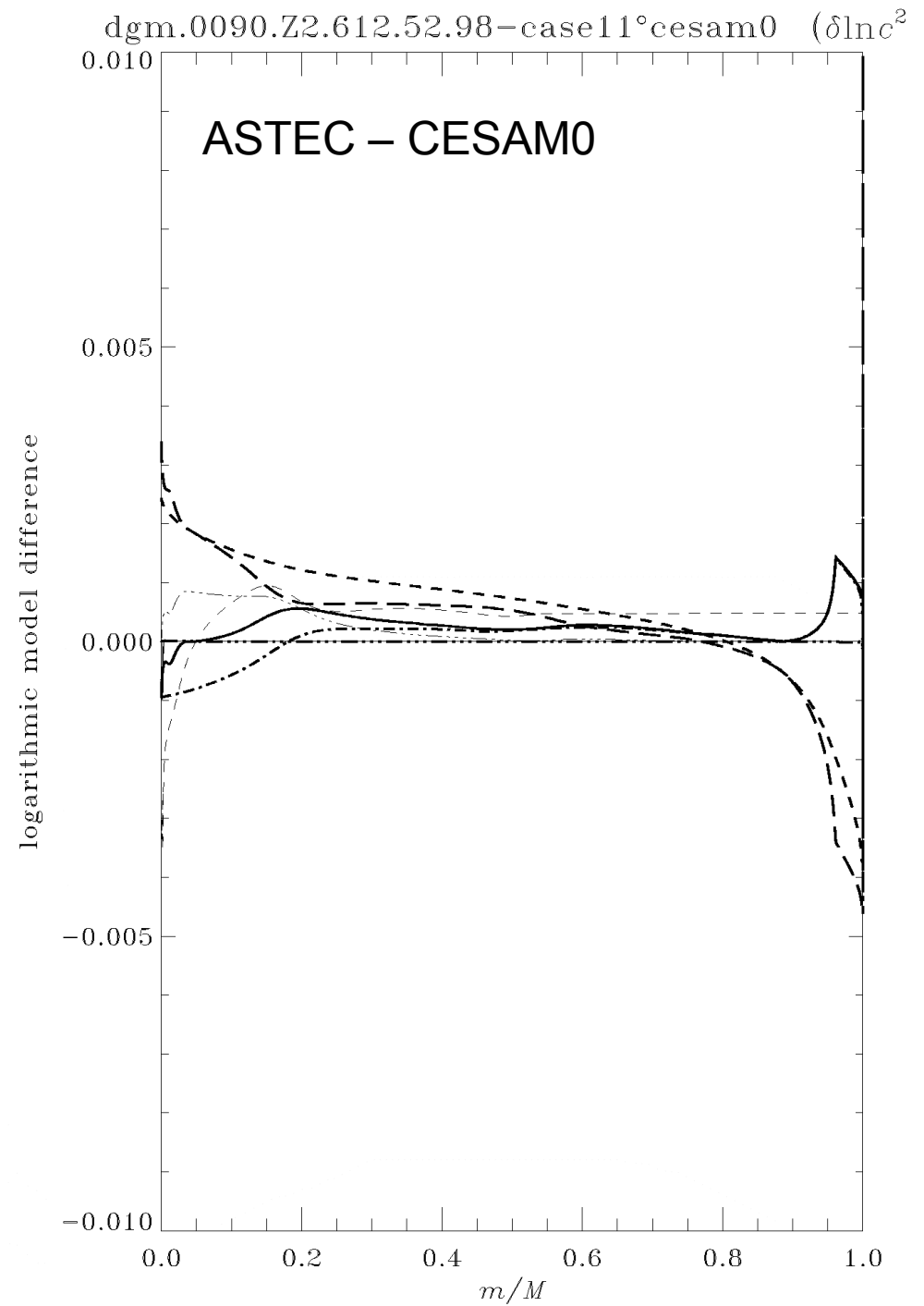
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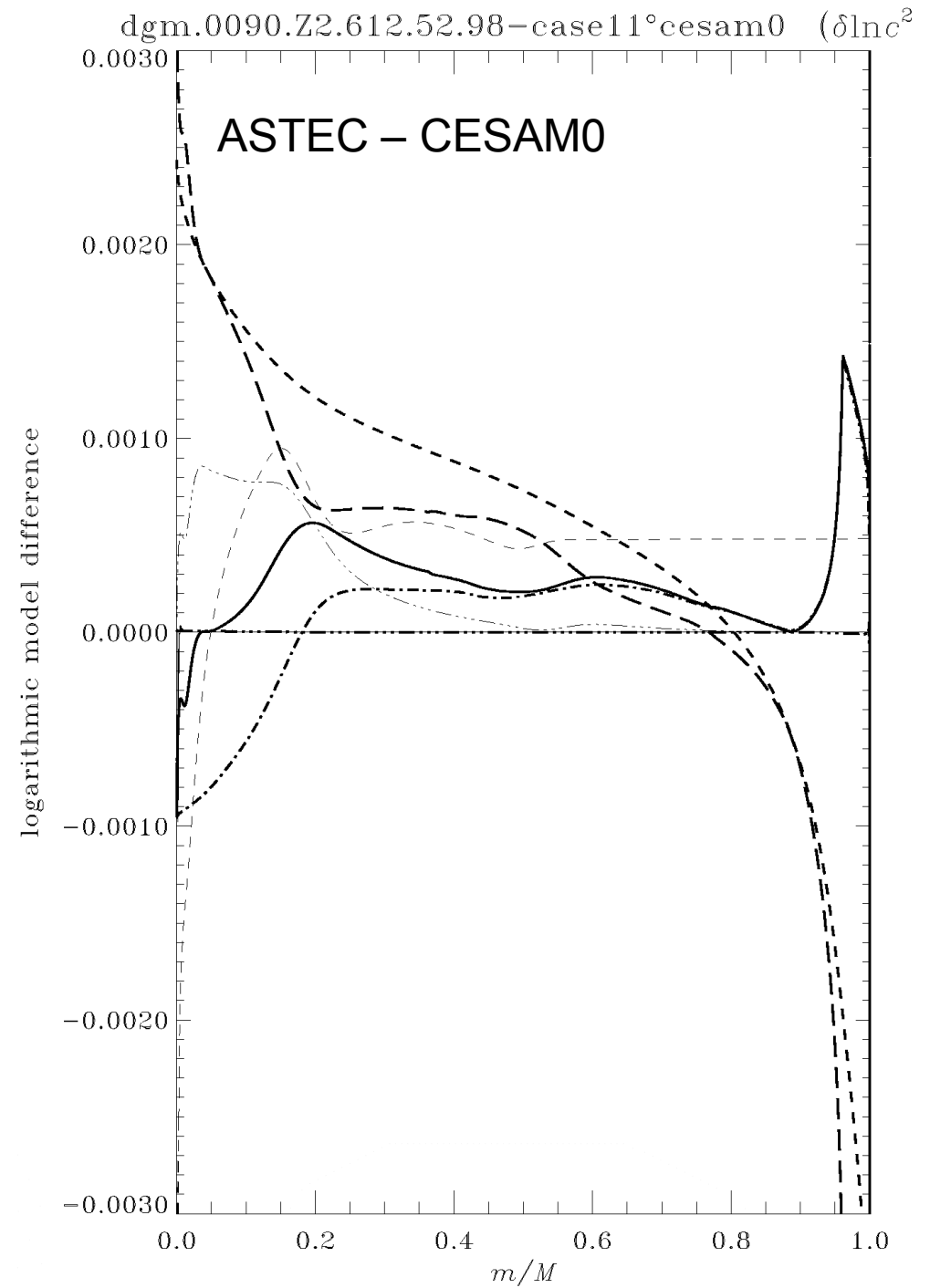
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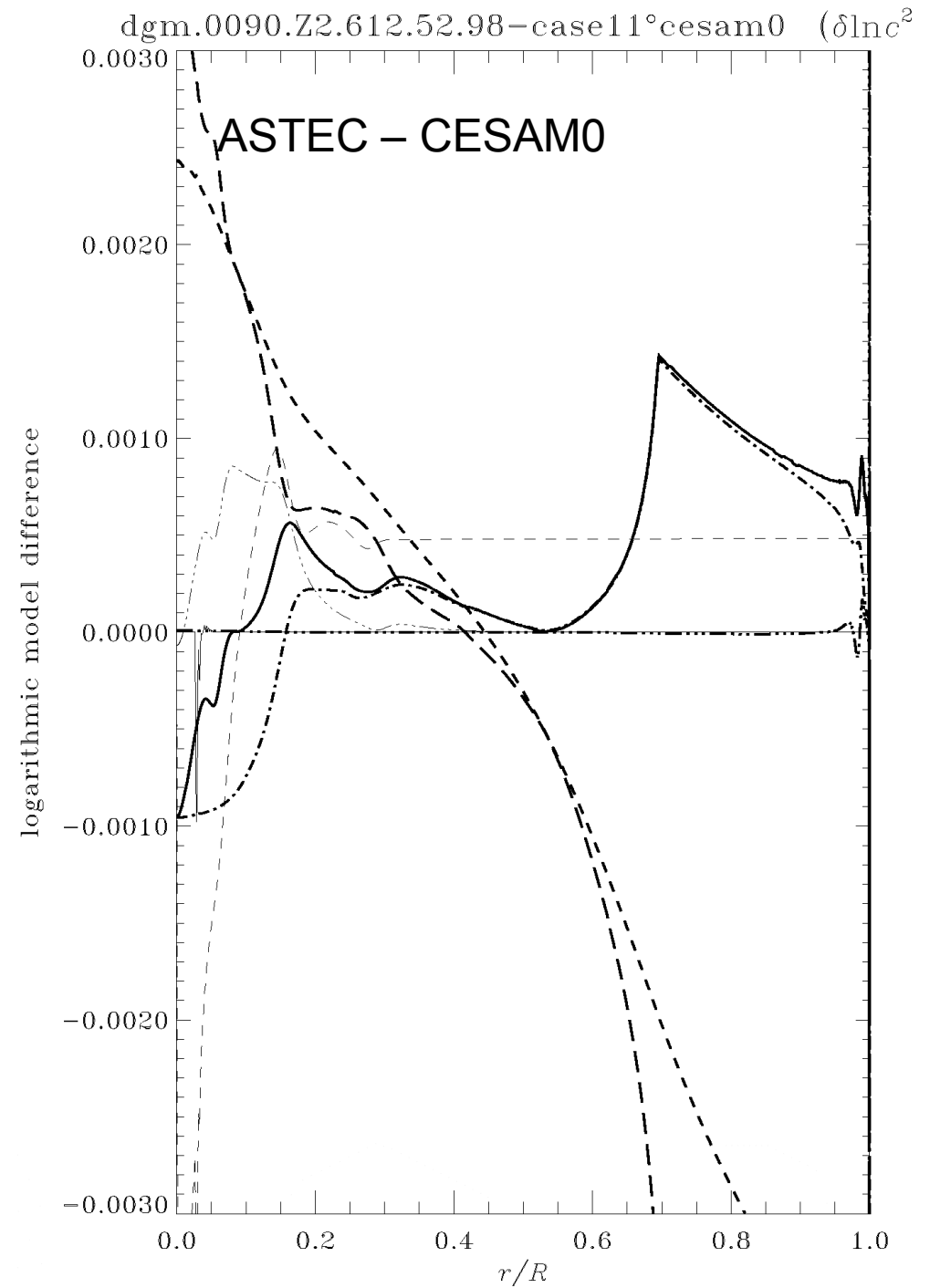
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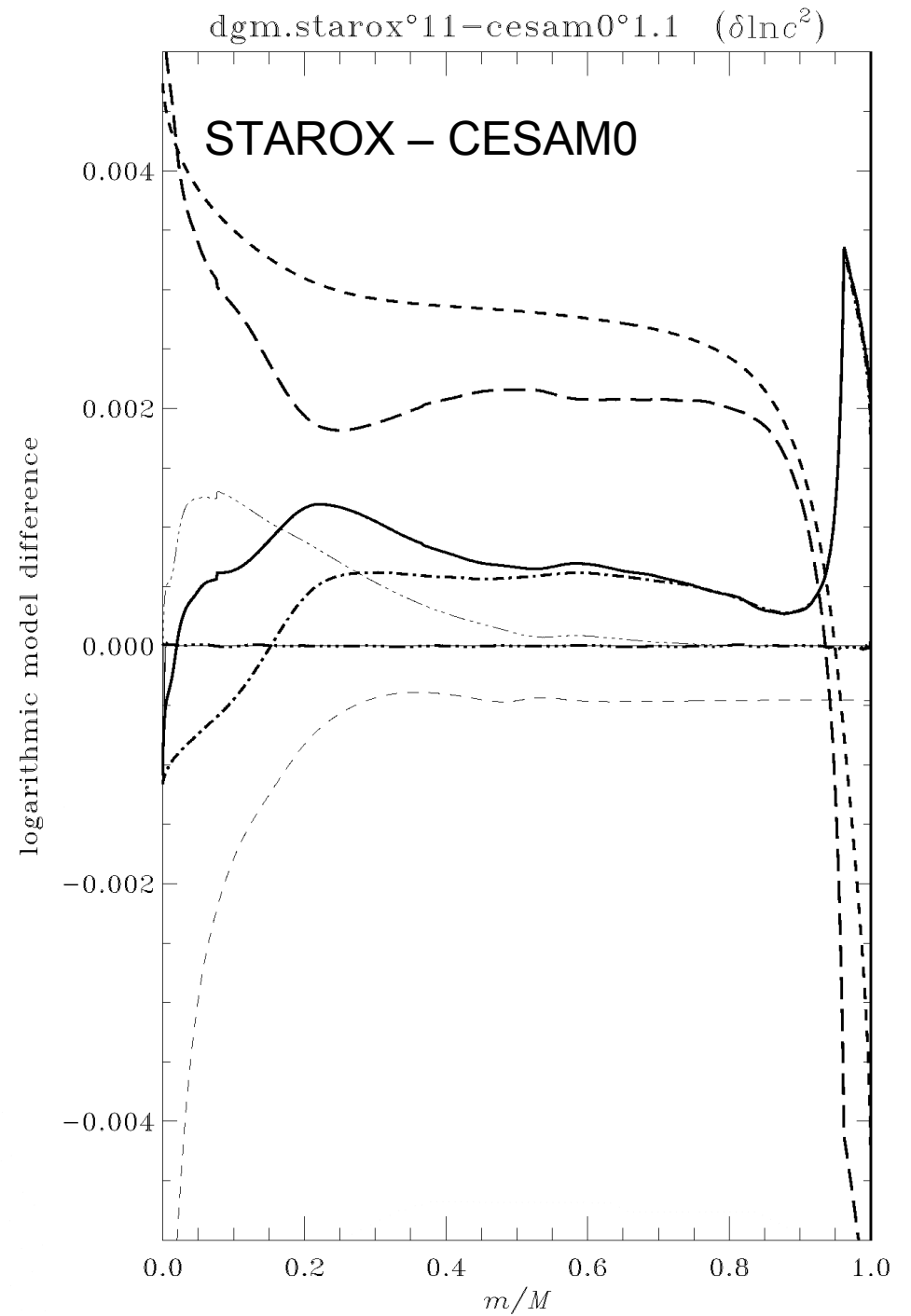
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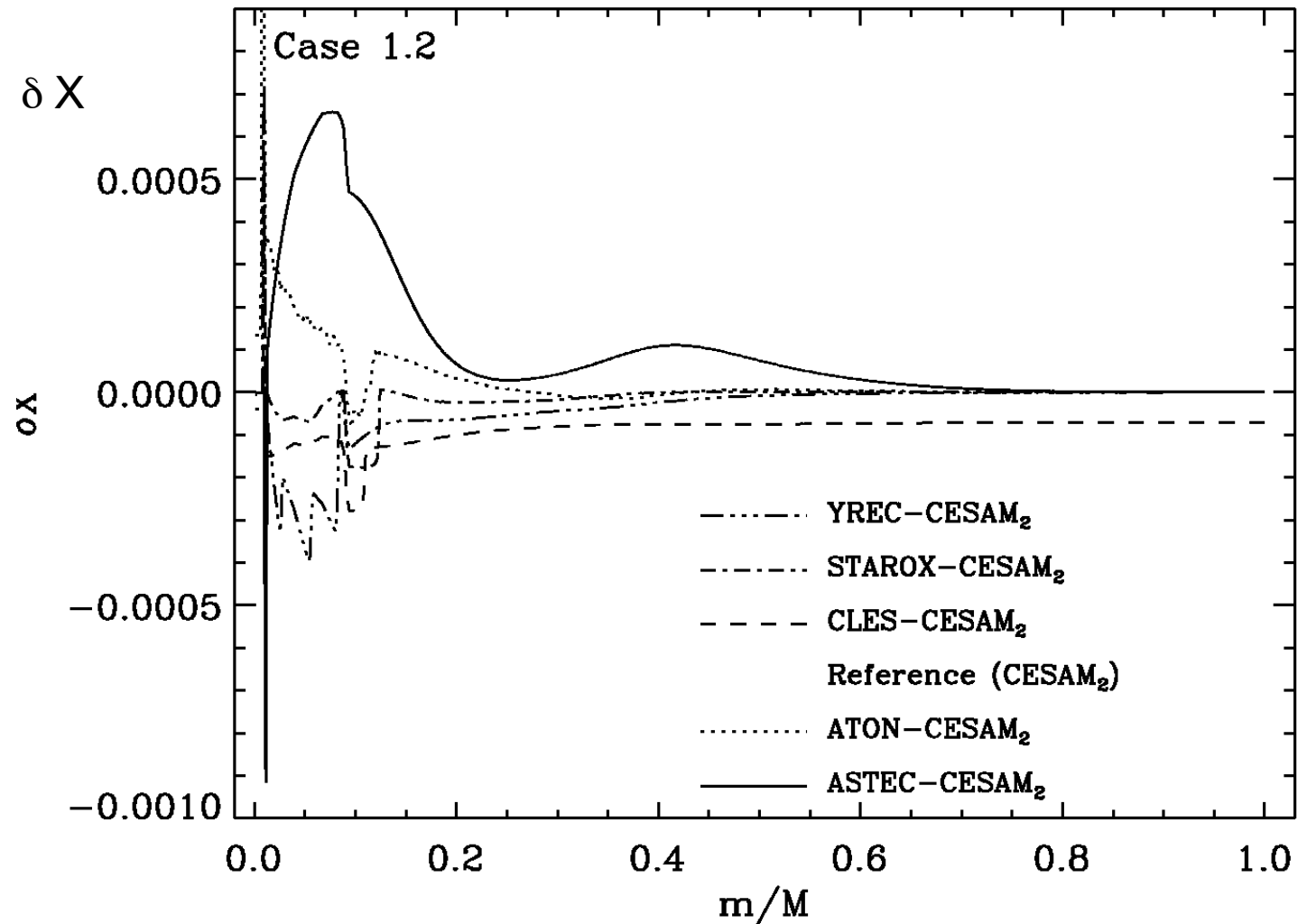
Hydrogen abundance

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$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$



Case 1.2

1.2 M-

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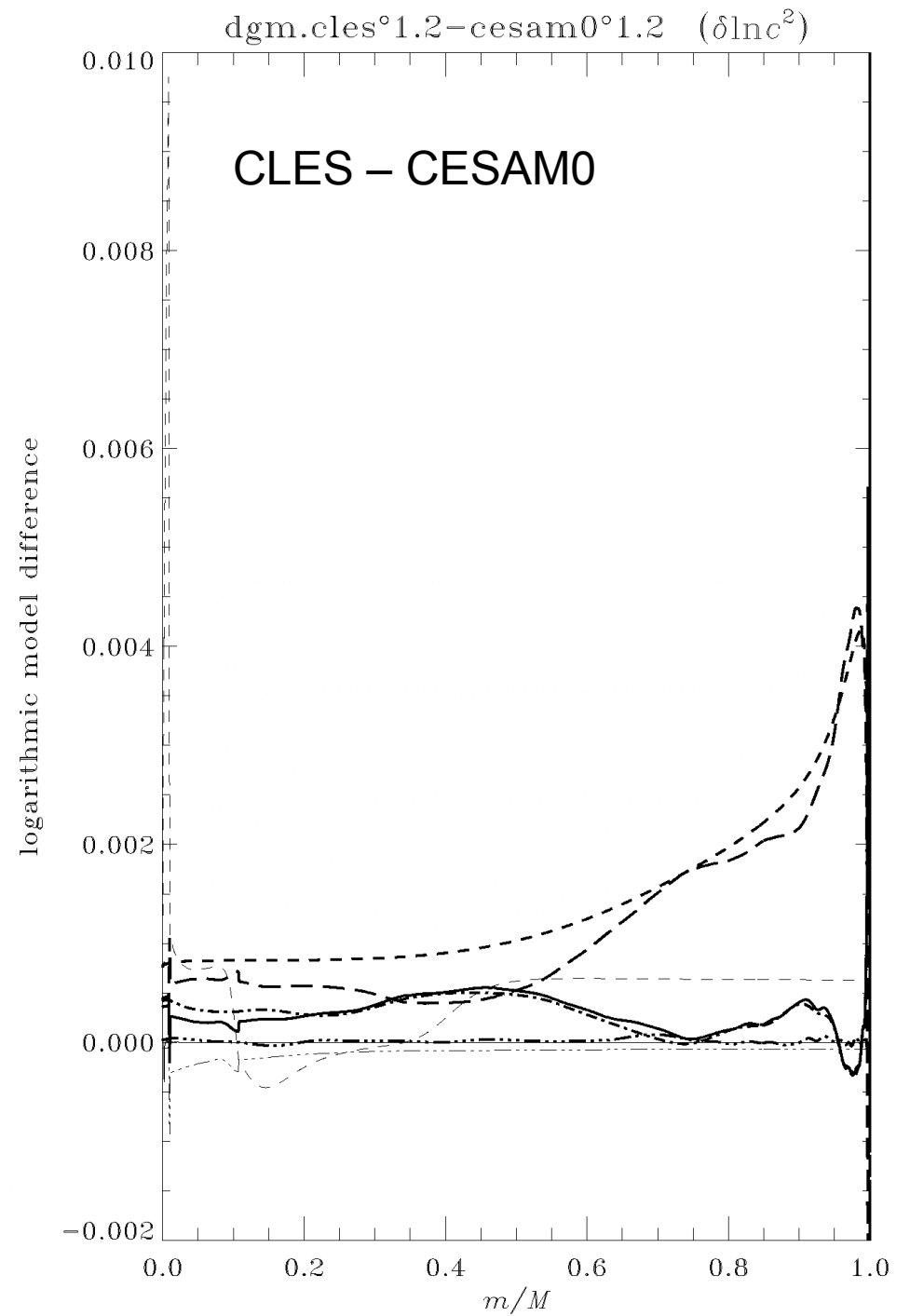
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Case 1.2

1.2 M-

$$X_0 = 0.7$$

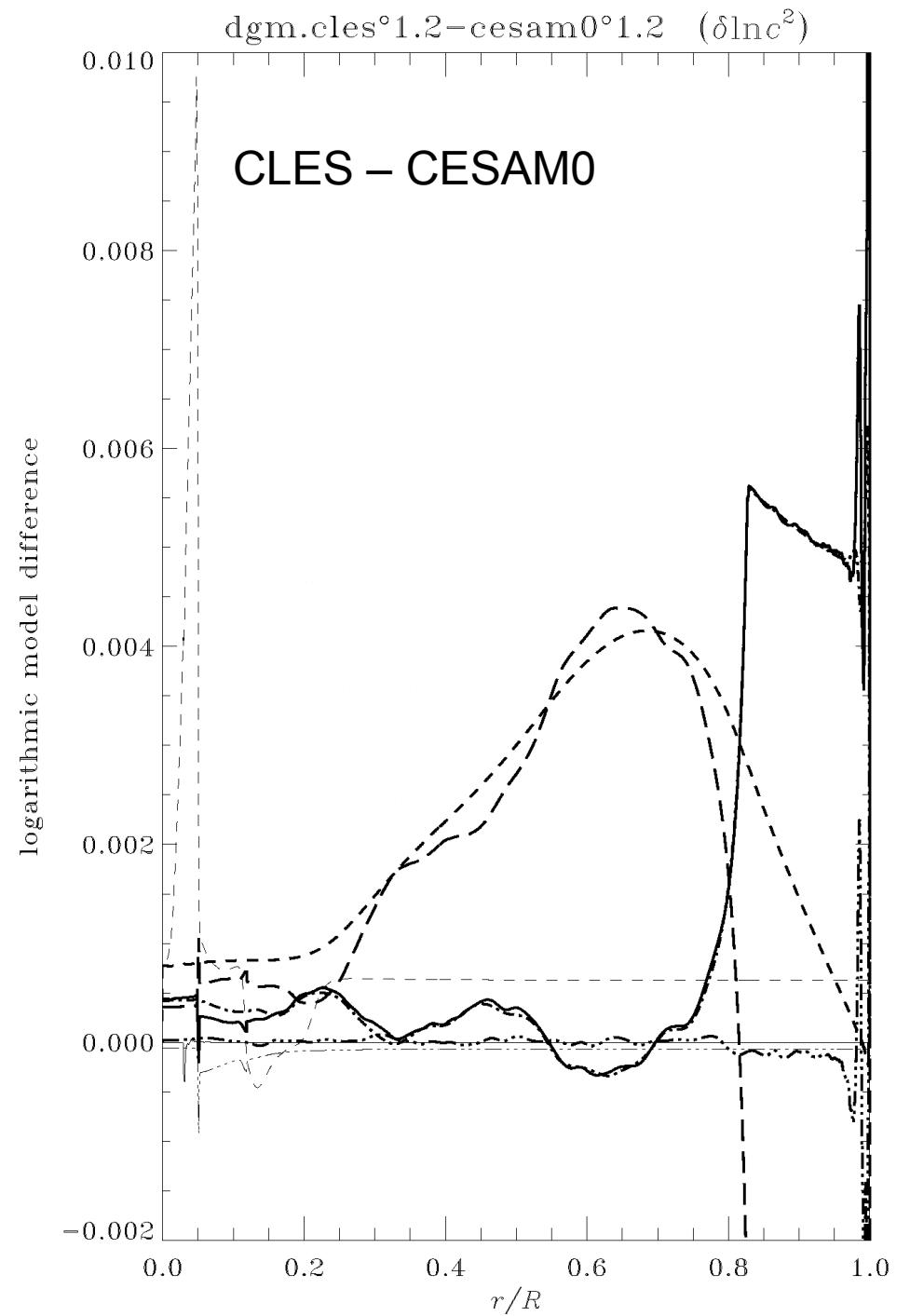
$$Z_0 = 0.02$$

$$X_c = 0.69$$

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
 $- - -$: $\delta \ln \rho$
 — : $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

— : $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX



Case 1.2

1.2 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

----- : $\delta \ln T$

----- : $\delta \ln p$

----- : $\delta \ln \rho$

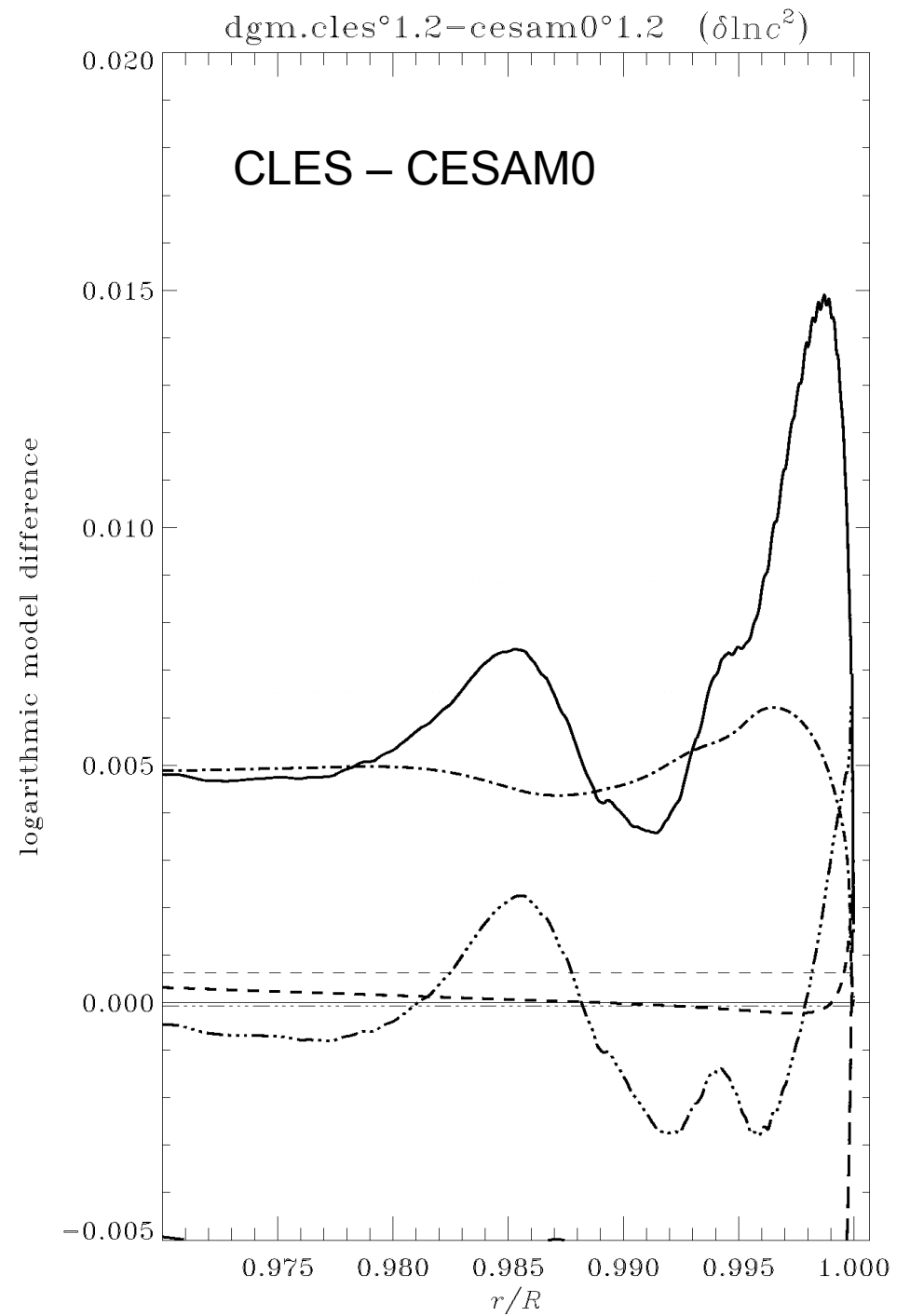
———— : $\delta \ln c^2$

..... : $\delta \ln \Gamma_1$

———— : $\delta \ln q$

----- : $\delta \ln L$

----- : δX



Case 1.2

1.2 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

..... : $\delta \ln T$

----- : $\delta \ln p$

----- : $\delta \ln \rho$

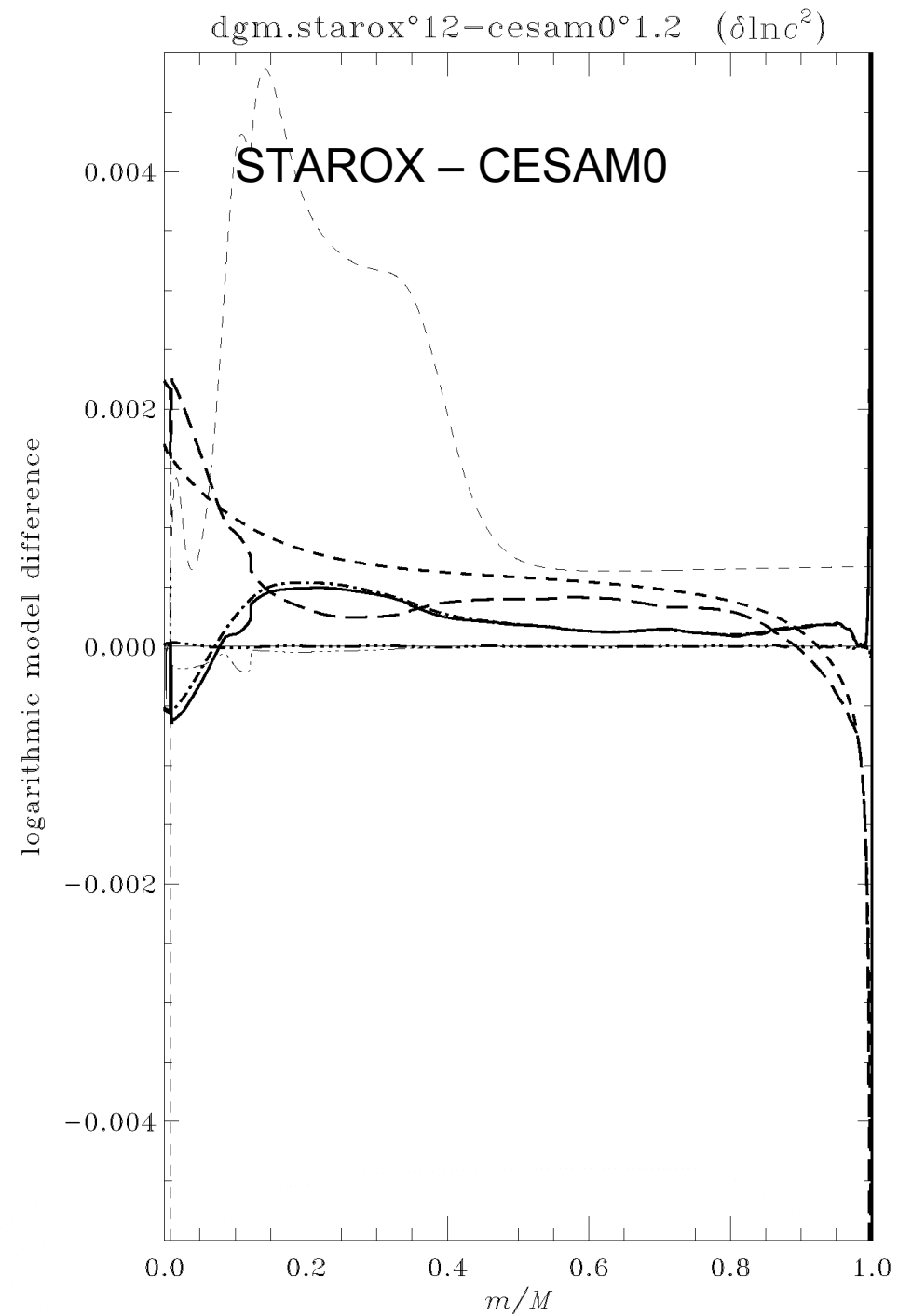
———— : $\delta \ln c^2$

-·-·-·-· : $\delta \ln \Gamma_1$

———— : $\delta \ln q$

----- : $\delta \ln L$

----- : δX



Case 1.2

1.2 M-

$X_0 = 0.7$

$Z_0 = 0.02$

$X_c = 0.69$

Line styles:

..... : $\delta \ln T$

----- : $\delta \ln p$

- - - - - : $\delta \ln \rho$

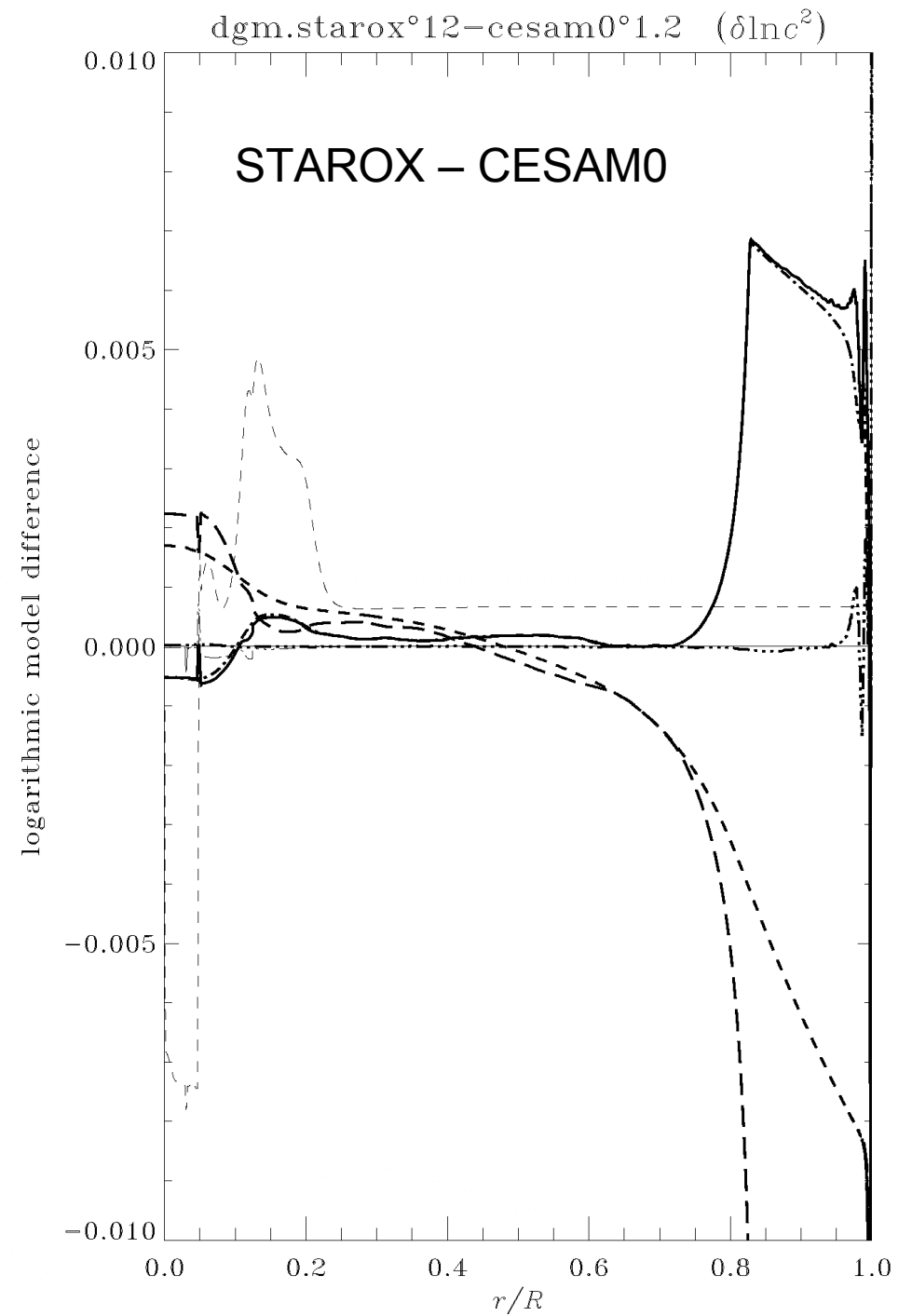
————— : $\delta \ln c^2$

- · - · - · : $\delta \ln \Gamma_1$

————— : $\delta \ln q$

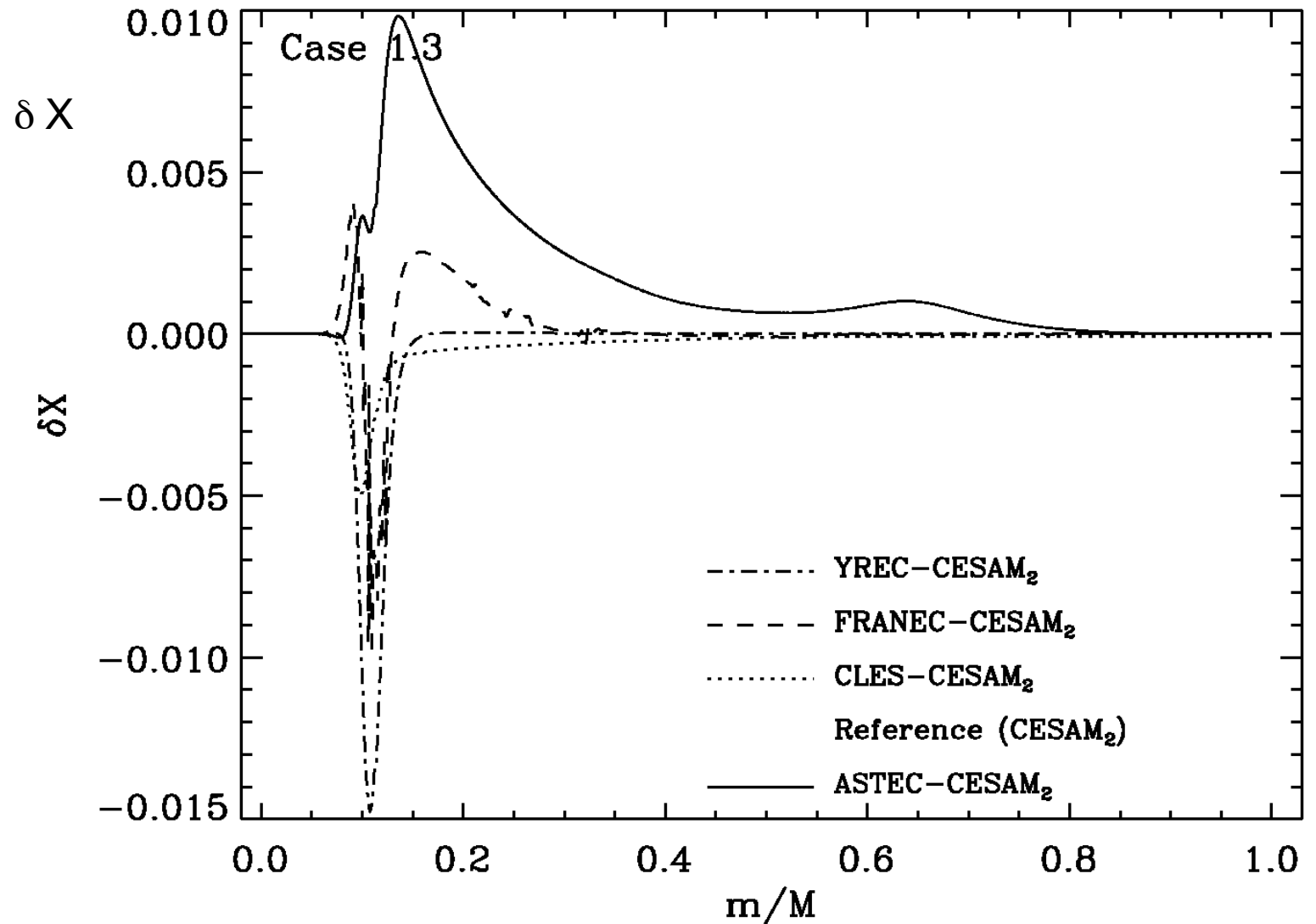
----- : $\delta \ln L$

- - - - - : δX



Hydrogen abundance

1.2 M-
 $X_0 = 0.73$
 $Z_0 = 0.01$
 $M_{\text{HeC}}/M = 0.1$



Case 1.3

1.2 M-

$X_0 = 0.73$

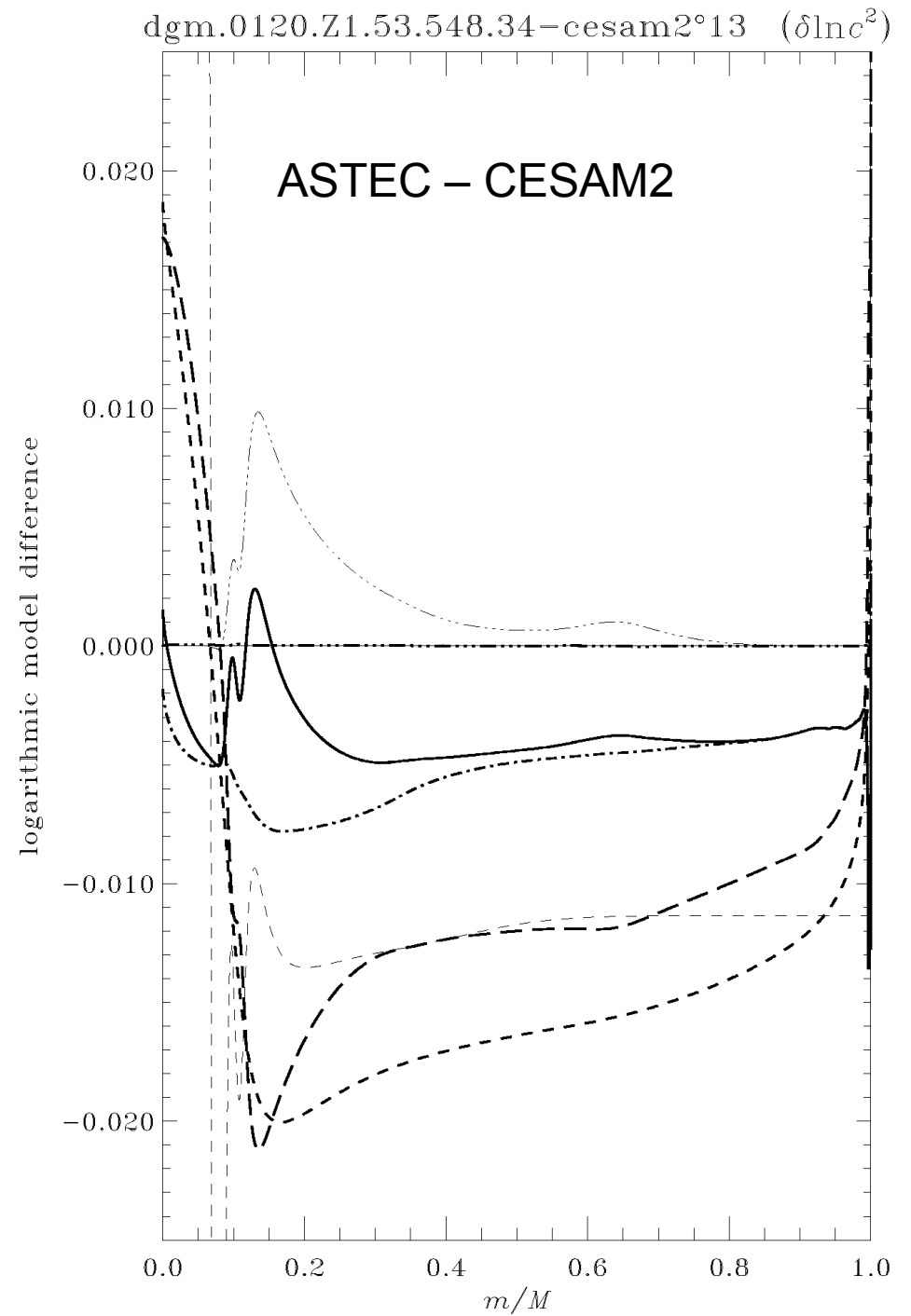
$Z_0 = 0.01$

$M_{\text{HeC}}/M = 0.1$

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
 $- - -$: $\delta \ln \rho$
 $---$: $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

$---$: $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX



Case 1.3

1.2 M-

$X_0 = 0.73$

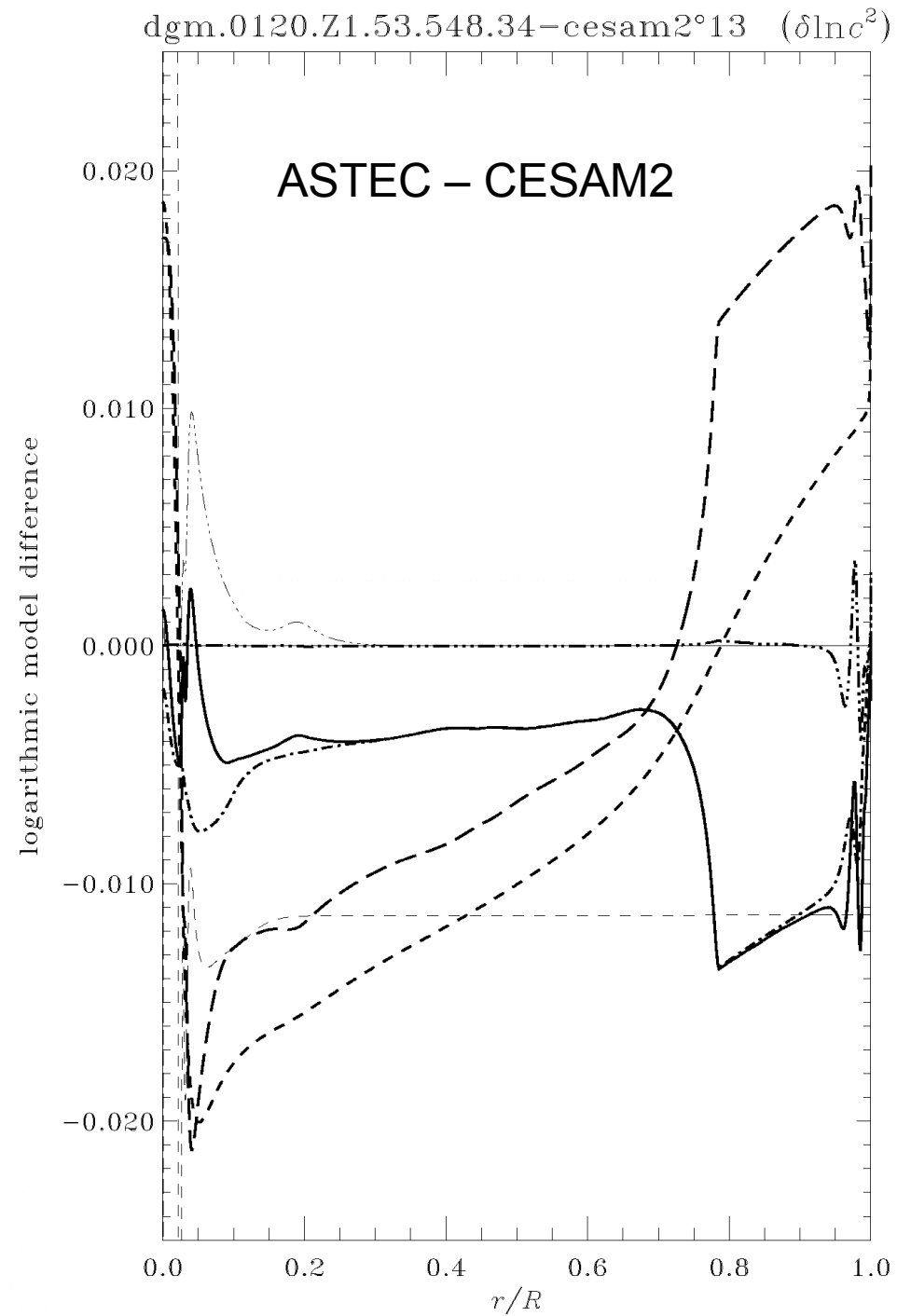
$Z_0 = 0.01$

$M_{\text{HeC}}/M = 0.1$

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
 $- - -$: $\delta \ln \rho$
 $---$: $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

$---$: $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX



Hydrogen abundance

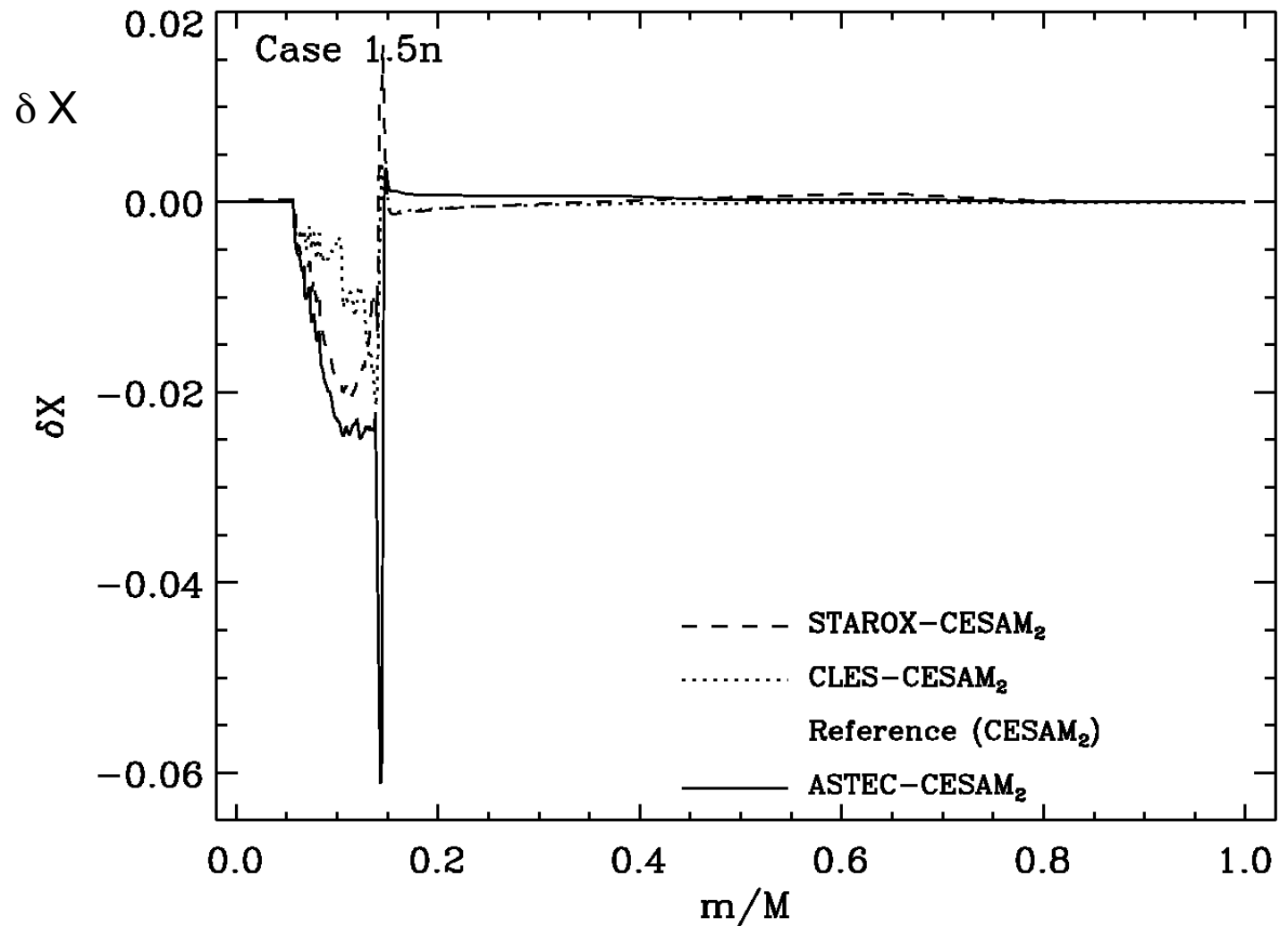
2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

No overshoot



Case 1.5n

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

No overshoot

Line styles:

..... : $\delta \ln T$

----- : $\delta \ln p$

----- : $\delta \ln \rho$

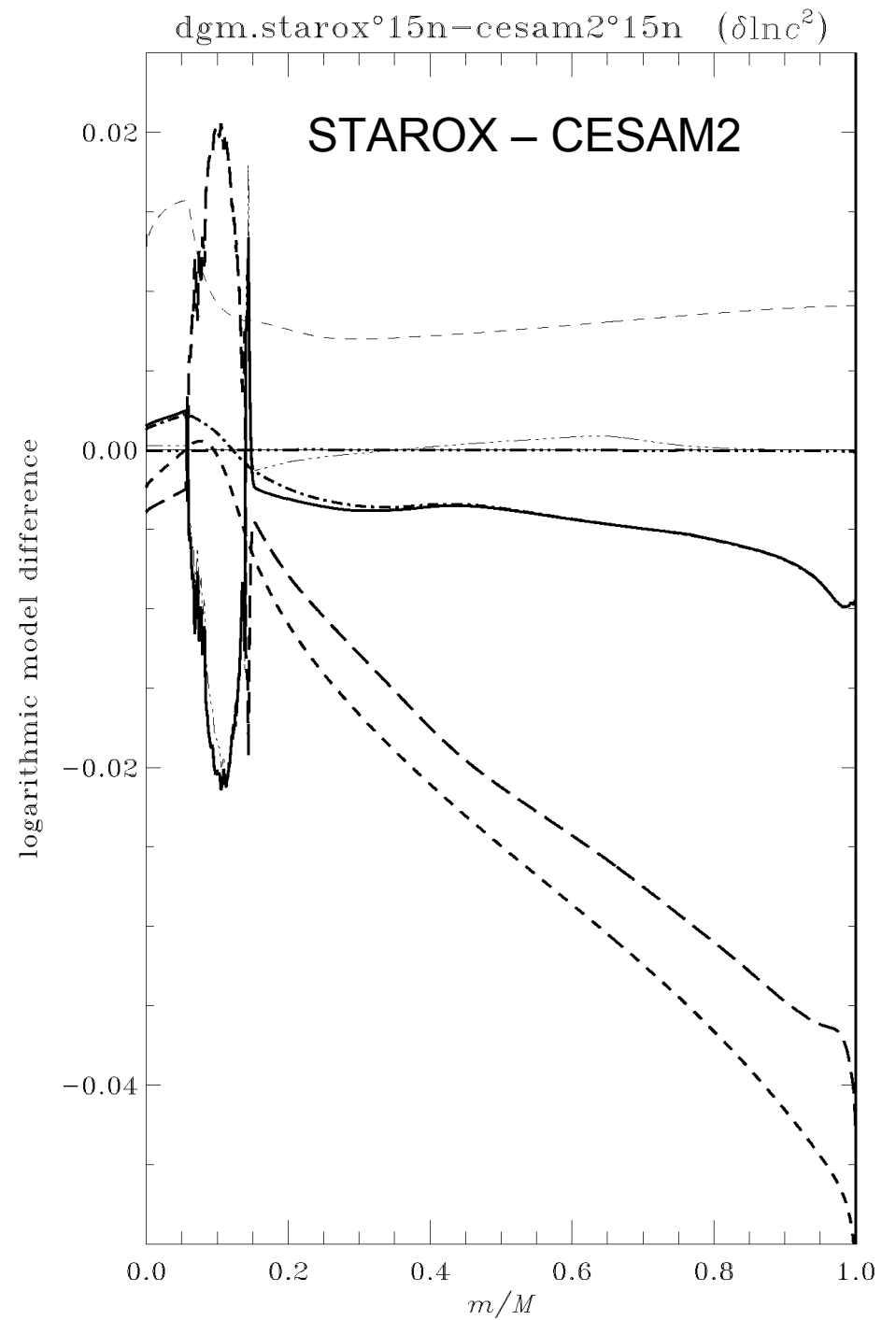
———— : $\delta \ln c^2$

..... : $\delta \ln \Gamma_1$

———— : $\delta \ln q$

----- : $\delta \ln L$

..... : δX



Case 1.5n

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

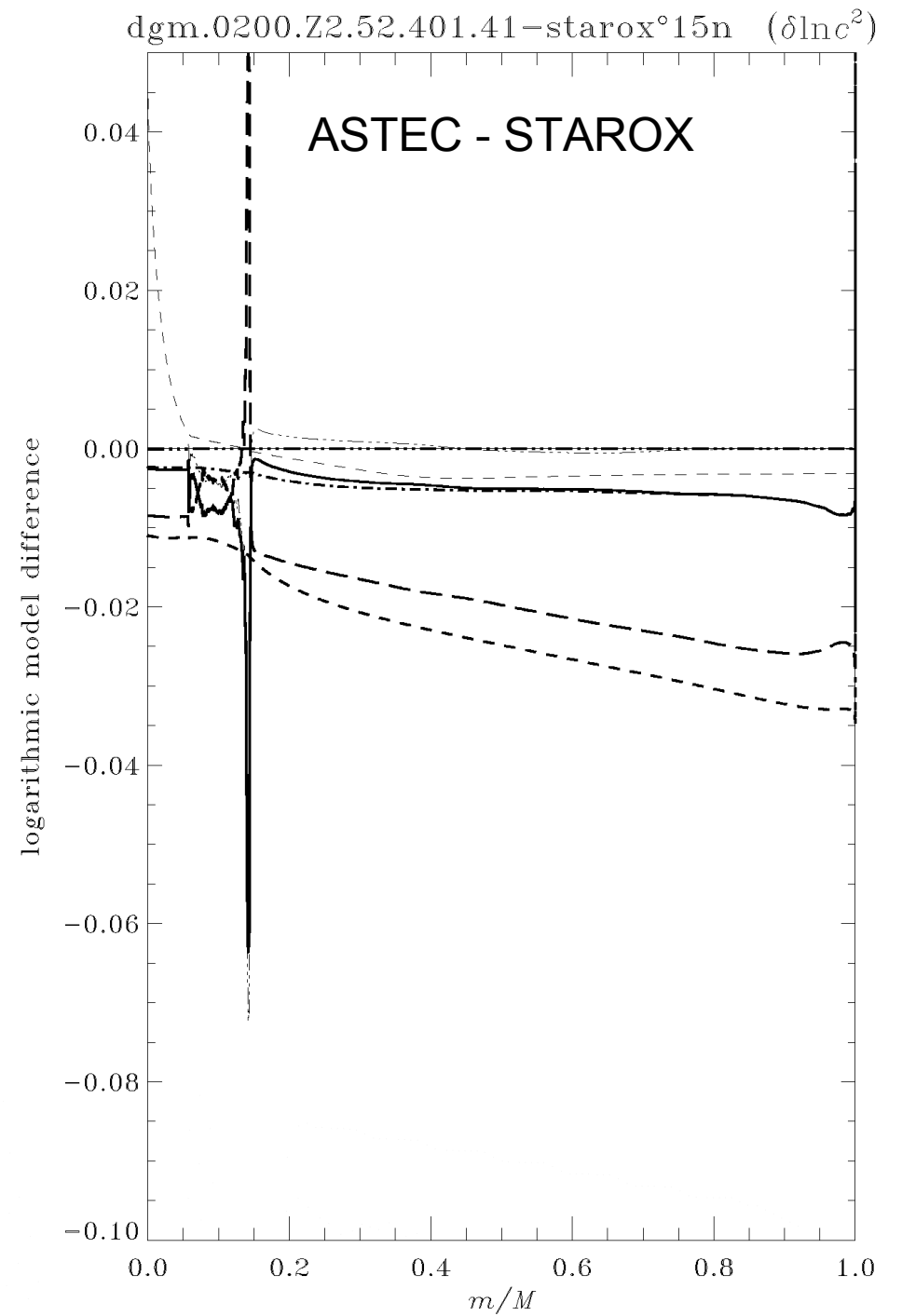
$X_c = 0.01$

No overshoot

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
 $- - -$: $\delta \ln \rho$
 — : $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

— : $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX



Hydrogen abundance

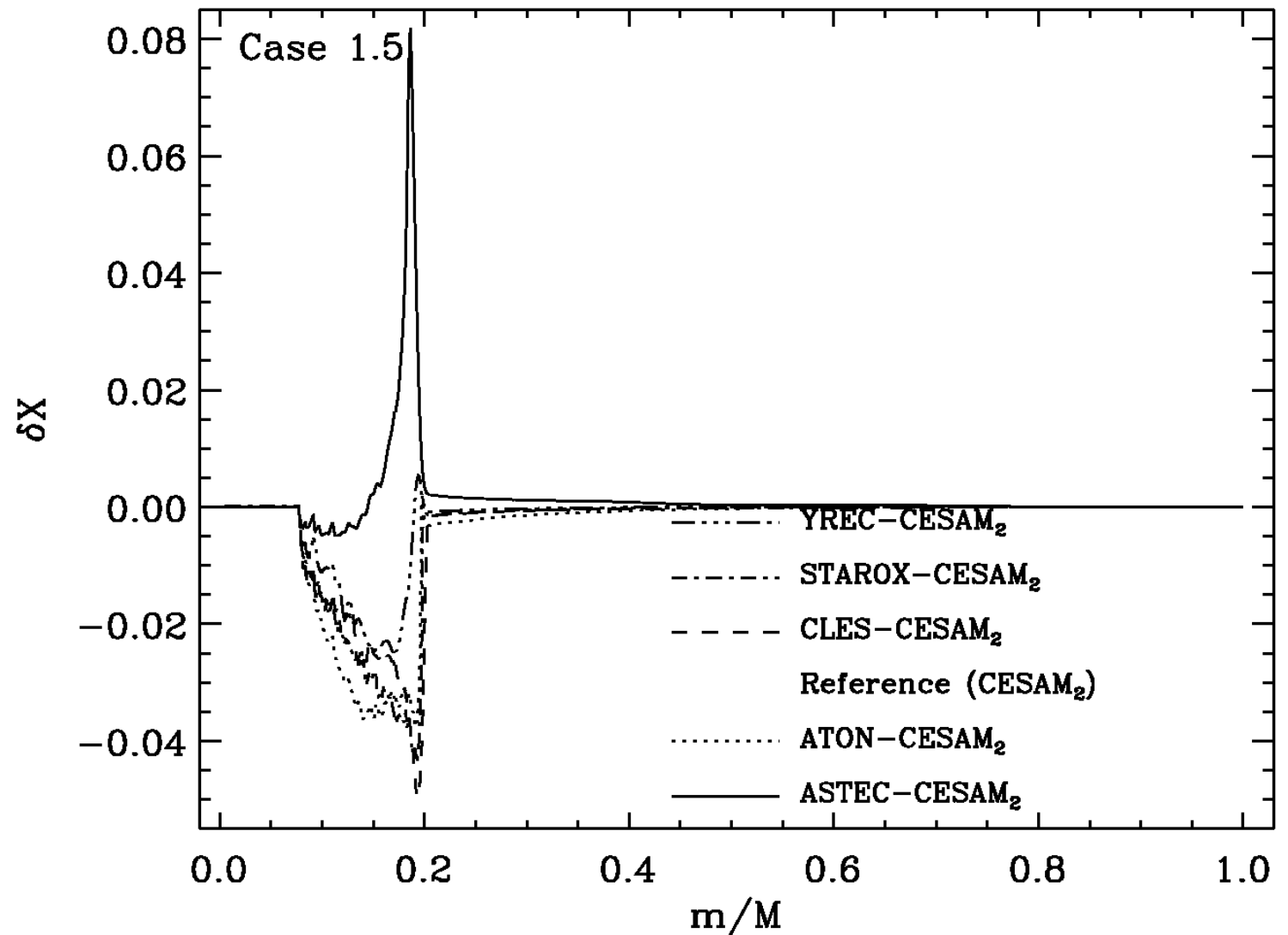
2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

Overshoot,
 $0.15 H_p$



Case 1.5

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

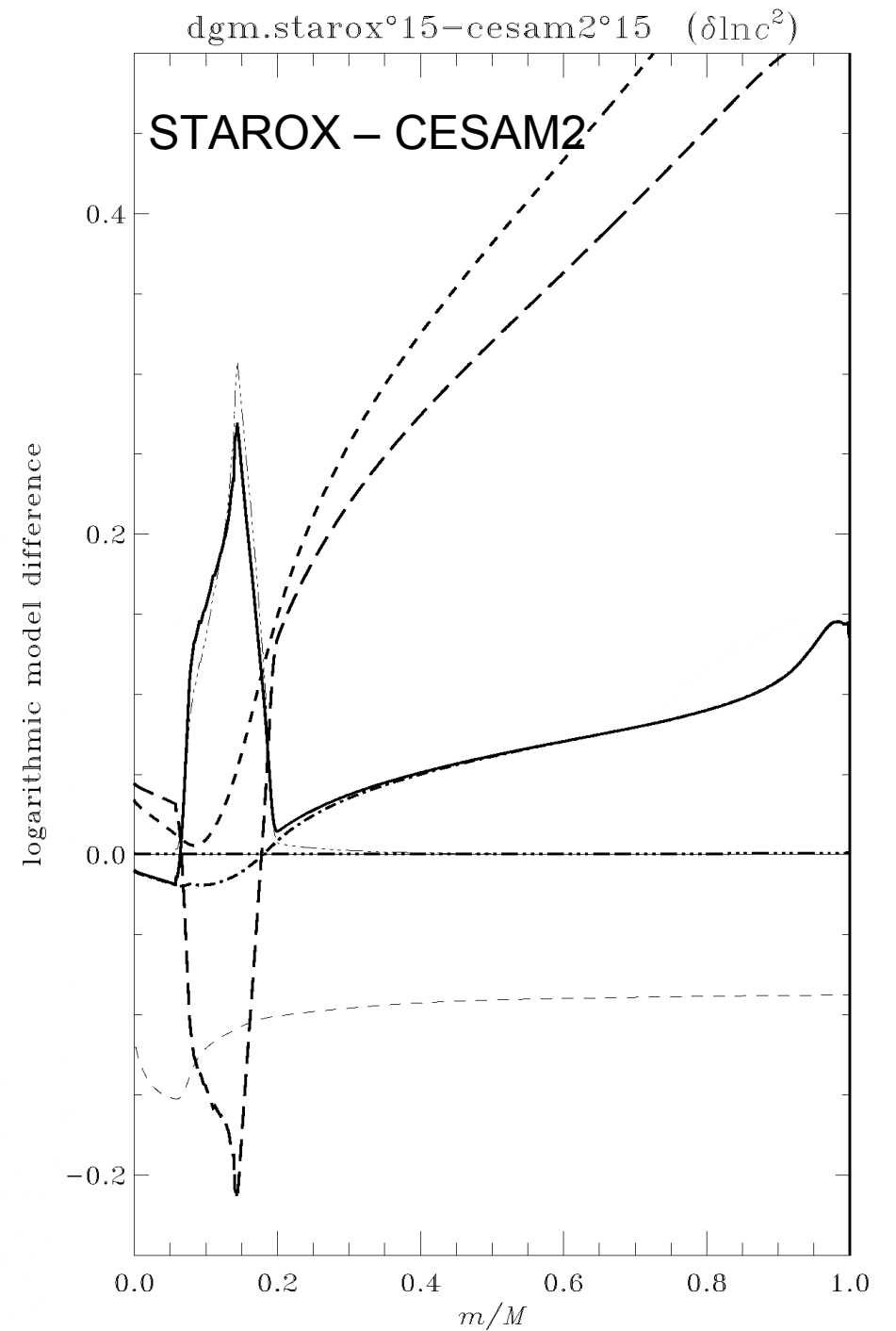
$X_c = 0.01$

Overshoot
0.15 H_p

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
 $- - -$: $\delta \ln \rho$
 $---$: $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

$---$: $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX



Case 1.5

2.0 M-

$X_0 = 0.72$

$Z_0 = 0.02$

$X_c = 0.01$

Overshoot
0.15 H_p

Line styles:

\cdots : $\delta \ln T$
 $---$: $\delta \ln p$
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 $---$: $\delta \ln c^2$
 \cdots : $\delta \ln \Gamma_1$

$---$: $\delta \ln q$
 $---$: $\delta \ln L$
 \cdots : δX

