ASTEC

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Basic numerical scheme

$$\begin{array}{ll} \frac{\partial z_l}{\partial x} \ = \ f_l(x;y_i;t) \,, & l = 1, \dots I_1 \quad \mbox{Type I} \\ \frac{\partial z_p}{\partial x} \ = \ f_p(x;y_i;t) + \sum_{i=1}^{l} \Lambda_{pi}(x;y_j;t) \frac{\partial y_i}{\partial t} \,, & p = I_1 + 1, \dots I_1 + I_2 \quad \mbox{Type II} \\ \frac{\partial z_u}{\partial t} \ = \ f_u(x;y_i;t) \,, & u = I_1 + I_2 + 1, \dots I_1 + I_2 + I_3 \quad \mbox{Type III} \\ z_i = z_i(x;y_j;t) \,, & i = 1, \dots I \,, \\ g_\alpha(x_1;y_i(x_1);\lambda_k) \ = \ 0 \,, & \alpha = 1, \dots, KA \,, \\ g_\beta(x_2;y_i(x_2);\lambda_k) \ = \ 0 \,, & \beta = KA + 1, \dots, KA + KB \,. \end{array}$$

Subroutine tnrkt

Treatment of diffusion

Basic equation

$$\frac{\partial X_i}{\partial t} = \frac{\partial}{\partial m} \left(\mathcal{D}_i \frac{\partial X_i}{\partial m} \right) + \frac{\partial}{\partial m} (\mathcal{V}_i X_i) + \mathcal{R}_i ,$$

introduce

$$Y_i = \mathcal{D}_i \frac{\partial X_i}{\partial m} + \mathcal{V}_i X_i$$
;

Then

$$\frac{\partial X_i}{\partial m} = \mathcal{D}_i^{-1} Y_i - \mathcal{D}_i^{-1} \mathcal{V}_i X_i ,$$

$$\frac{\partial Y_i}{\partial m} = \frac{\partial X_i}{\partial t} - \mathcal{R}_i ,$$

is on tnrkt form

Implementation details: equation of state

OPAL 2001 tables, for the appropriate value of Z (= 0.02 or 0.01). Using the OPAL interpolation scheme.

Implementation details: opacity

OPAL 1996 tables, with Alexander low-temperature values. Houdek interpolation scheme. The heavy-element abundance is taken to be the initial value, regardless of the changes due to nuclear reactions.

(In models with diffusion and settling, these effects on heavy elements are decoupled from the nuclear changes in composition, for now. The modified heavy-element abundance is used for the opacity but not, usually, for interpolated equations of state.)

Implementation details: nuclear reactions

For Toulouse comparison: nuclear reaction parameters generally from Bahcall & Pinsonneault (1995).

NACRE now implemented.

Salpeter weak screening.

Implementation details: treatment of convection

Böhm-Vitense (1958) mixing-length treatment, probably with Henyey et al. (1965) detailed parameters. (The specification of these parameters seems a little uncertain.)

Turbulent pressure is not included.

Implementation details: treatment of atmosphere

Integration of hydrostatic equation assuming the grey Eddington T(τ) relation: T = T_{eff} [3/4*(τ + 2/3)]^{0.25}, starting at τ = 0.01 and matching where T = T_{eff} (τ = 2/3).

Note that there are potential problems with the treatment of radiation pressure in the atmosphere, certainly for relatively massive (and hot) stars.)

Detailed test of Case 1.1



Implementation details: initial model, chemical evolution

The models start from the ZAMS (PMS evolution has been used in the code, but not tested with care).

Chemical evolution is integrated with the general solution of the structure and evolution equations (tnrkt).

A separate treatment is used for the chemical evolution of the convective core, using the averaged reaction rates. This is carried out in parallel with the Henyey iteration, although occasionally with fixes to ensure convergence (freezing the properties of the core).

CN part of the CNO cycle is assumed to be in nuclear equilibrium at all times; initial ¹⁴N abundance includes also the original ¹²C abundance. The conversion of ¹⁶O into ¹⁴N is taken into account. The initial abundances of ¹⁴N and ¹⁶O, relative to the heavy-element abundance, are 0.2337 and 0.5154. (May well need to be changed, to meet specifications.)

Implementation details: treatment of ³He

Original models (for Toulouse):

³He is assumed to be in nuclear equilibrium at all times.

Recent modelling (and always used in solar case):

Set ZAMS ³He abundance by evolving abundance for time τ_3 at constant conditions, starting from zero abundance.

Typical value (for solar models): $\tau_3 = 5 \times 10^7$ years

Also try τ_3 = 10^7 years for 0.9 M_{\odot} model.

$$0.9 \text{ M}_{\odot}, \text{ X}_{c} = 0.35$$

(³He evolution, $\tau_3 = 10^7$ years) – (³He equil.)

Note: early small convection core.

: $\delta \ln q$

: $\delta \ln L$

 $: \delta X$



Line styles:



³He abundance



Implementation details: Mesh

The scheme for defining the mesh is broadly as described by CD82; however, a very dense mesh is used near the boundary of a possible convective core.

Most calculations for CoRoT comparison used 601 points (between centre and photosphere).

Mesh reset for pulsation calculation, depending on desired modes (distribution largely determined by asymptotic behaviour of eigenfunctions). $1.3~M_{\odot}$



 $1.3~M_{\odot}$



 $1.3~M_{\odot}$



 $1.3~M_{\odot}$



 $1.3~M_{\odot}$



Case 1.1

$$0.9 \text{ M}_{\odot}, \text{ X}_{c} = 0.35$$

³He in equilibrium

Test effect of no. of meshpoints: (N = 1200) - (N = 600)



Line styles:



 $-:\delta \ln q$

: $\delta \ln L$

 \cdots : δX

Implementation details: Timestep

The timestep is set based on relative (or \log_{10}) changes in several quantities being limited to be below a specified limit Δy_{max} .

Changes in the hydrogen abundance in a convective core are scaled by a factor 5, to compensate for the rather crude numerical treatment of the core composition. As a result, more timesteps are used in models with a convective core.

In the present case, typically 200 steps are required to reach exhaustion of hydrogen at the centre, in models with a convective core, and 30 - 40 steps in a model without (this small number is also a consequence of the crude treatment of the nuclear network). Case 1.1

$$0.9 \ M_{\odot}, \ X_{c} = 0.35$$

³He in equilibrium

Test effect of no. timesteps:

 $(N_t = 24) - (N_t = 13)$ $(\Delta y_{max} = 0.025) - (\Delta y_{max} = 0.05)$



Line styles:



 $\begin{array}{c} ----- : \ \delta \ln q \\ ---- : \ \delta \ln L \\ ---- : \ \delta X \end{array}$