A Numerical Comparison of Stellar Oscillation Codes

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I. Introduction

The eigenfrequencies and the eigenfunctions of oscillations in stars can be obtained as solutions of a set of equations derived from the basic equations of stellar structure. This makes it possible to calculate these solutions numerically by applying the oscillation equations to an equilibrium model of a star. Such stellar oscillation codes have been developed over the years by numerous researchers in the fields of helio- and asteroseismology. In the context of the ongoing research for the forthcoming asteroseismic missions, including COROT, it might be worthwhile to compare the results of these codes in order to estimate the uncertainties even in a purely theoretical exercise and to devise possible ways to refine their accuracy. In the present exercise, we compare the eigenfrequencies generated by four stellar oscillation codes developed independently by different authors. It must be stressed that we consider only the numerical agreement, or lack thereof, between these codes as applied to the same stellar models under identical boundary conditions. Comparison of the relative performance of the codes in terms of speed, absolute accuracy of results or flexibility and range of application is outside the scope of this study.

II. The Oscillation Codes

The general scheme of the exercise is to run the different codes with the same input equilibrium stellar model to calculate the eigenfrequencies of low degree p-modes of oscillation. The boundary conditions are identical in each case, being that the Lagrangian pressure perturbation vanishes at the outer boundary. Only frequencies less than the acoustic cutoff are used for comparison.

We compare four stellar oscillation codes developed by different authors. All the codes use the adiabatic approximation. We distinguish them according to the authors and give a brief description of each below.

1. By J. Christensen-Dalsgaard (Aarhus), known as ADIPLS

The ADIPLS code (http://astro.ifa.au.dk/~jcd/adipack.n) has been used as the primary template for the frequencies, i.e., the frequencies obtained from all the other three codes are compared to the frequencies computed by ADIPLS.

The ADIPLS code provides two sets of solutions for each model — frequencies calculated by application of the variational principle and frequencies corrected for truncation errors through Richardson extrapolation. We denote these frequencies by $\nu_{J(V)}$ and $\nu_{J(R)}$, respectively. One major difference of the ADIPLS code with the other three is that while it solves the full fourth-order set of equations for non-radial oscillations, in the radial case the perturbation in the gravitational potential is eliminated analytically and a second-order set of equations are used. It is possible that this is the reason behind the somewhat different behaviour of $\ell = 0$ modes of this code with respect to the other codes as compared to non-radial modes.

2. By I. W. Roxburgh & S. V. Vorontsov (QMUL)

This code does not employ the Richardson extrapolation technique. Instead, it solves the equations through a fourth-order Runge-Kutta method, which requires an interpolation of the supplied model to produce a finer grid of mesh points. We denote the frequencies from this code by ν_R.

3. By H. M. Antia (TIFR)

Antia's code uses the Richardson extrapolation method to produce the final set of frequencies. These frequencies are denoted by ν_A.

4. By L. Léon & F. Tran Minh (Meudon), known as FILOU

Although the FILOU code (Tran Minh & Léon 1995) does not apply the Richardson technique, the frequencies used in this exercise were actually corrected through this extrapolation method by running the code on the full grid and a coarser grid of alternate mesh points. We refer to these frequencies by ν_L.

III. The Models

Three stellar models were primarily used in this study, although several other models were used to check various aspects of the differences between the output from the codes. It is sufficient to focus here on these three models only to highlight the salient features of the analysis. We are guided in our choice of these models by the desire to test the codes on a fairly regular and smooth model (Model A below) as well as on models which might have some irregular features due to numerical difficulties encountered especially at the edges of convective regions (Models B and C).

A. Solar model — "Model S"

The first model that we consider is a solar model, described by Christensen-Dalsgaard et al.(1996) which has been extensively used as a reference for helioseismic inversion. The model uses OPAL equation of state and opacity, and includes diffusion of helium and heavy elements. This model provides the necessary quantities for the oscillation codes on 2482 mesh points, including the atmosphere.

B. CESAM model of mass 1.2 $M_\odot$

The second model is a stellar model of mass 1.2 $M_\odot$ and central hydrogen abundance of 0.4, generated by the CESAM evolutionary code (Morel 1997). OPAL equation of state and opacity was used to build this model. Diffusion of helium and heavy elements and convective overshoot were not considered. This model has a mesh size of 2099 points, including the atmosphere. This model has certain irregularities in the Brunt-Väisäla frequency profile at the edge of the convective core due to the sharp change in the density gradient.

C. CESAM model of mass 1.5 $M_\odot$

Finally, we consider another stellar model of mass 1.5 $M_\odot$ and central hydrogen abundance of 0.5, which is also generated by the CESAM evolutionary code and contains the same input physics as the second model described above. The structure of this model near the edge of the convective core is much more regular than the previous model.

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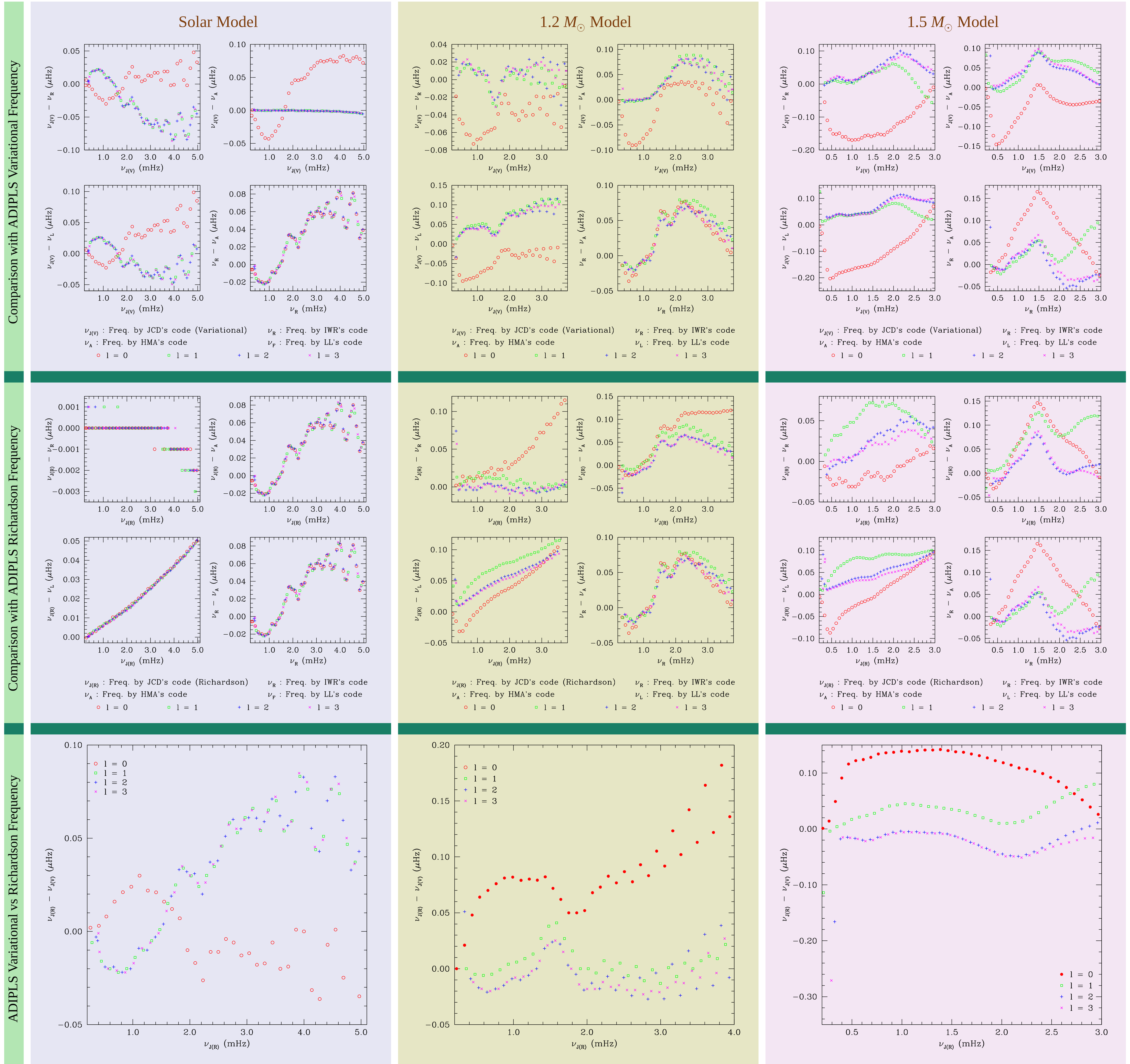

Figure 1: The nine figures above illustrate the main results of this study. The three columns refer to the three models — the solar model, the $1.20 M_\odot$ and the $1.50 M_\odot$ models, from left to right respectively. In each column, the top figure shows the comparison of the frequencies obtained from Roxburgh's, Antia's and the FILOU code against the variational frequencies obtained from Christensen-Dalsgaard's ADIPLS code. The middle figures show the comparison with respect to the Richardson frequencies obtained from ADIPLS. The bottom figures show the difference between the variational and the Richardson frequencies. In each figure of the top two rows, the top left panel plots the difference between ADIPLS and Roxburgh frequency against the ADIPLS frequency, the top right panel plots the difference between ADIPLS and Antia frequency against the ADIPLS frequency, the bottom left panel plots the difference between ADIPLS and FILOU frequency against the ADIPLS frequency, and the bottom right panel plots the difference between Roxburgh and Antia frequency against the Roxburgh frequency. The different modes of oscillation are distinguished by separate colours and symbols.

IV. Results

The main results of this comparative study are illustrated in Figure 1. We summarise them for each of the three models considered.

A. Solar model — "Model S"

For the solar model, the agreement between the five sets of frequencies, $\nu_{J(V)}$, $\nu_{J(R)}$, ν_R, ν_A and ν_L is typically within $0.1\,\mu$Hz. Specifically, ν_A agrees very well with the ADIPLS variational frequency, except for the radial mode, while ν_R has excellent agreement (down to the output accuracy of the codes) with the ADIPLS Richardson frequency. In fact, the third figure of this set (bottom left corner) shows that there is a difference of up to $0.08\,\mu$Hz between the variational and the Richardson frequencies, both calculated by ADIPLS. At higher frequencies, there also seems to be an oscillatory pattern in these differences. The difference between the variational and the Richardson frequency actually indicates the departure of the model from perfect hydrostatic equilibrium. Another feature which should be noted in these figures is the distinctly different pattern in the differences of the $\ell = 0$ mode compared to the non-radial modes (see e.g., $\nu_{J(V)}$ vs ν_A comparison). This occurs nearly for all comparisons with the ADIPLS frequencies for this model as well as the other two models and might be due to the fact that the radial mode equations are treated differently in ADIPLS, as described above.

B. CESAM model of mass 1.2 $M_\odot$

The magnitude of the differences between the frequencies for the $1.20 M_\odot$ model are generally higher than that of the solar model, although the pattern is similar. The differences typically increase with frequency, and the maximum deviation is $\sim 0.12\,\mu$Hz. Again, the ADIPLS radial mode behaves slightly differently. The discrepancy between the variational and the Richardson frequencies are up to $0.18\,\mu$Hz in this case, which might indicate a greater hydrostatic imbalance in the model. This is not surprising, given the structural irregularities near the edge of the convective core in this model.

C. CESAM model of mass 1.5 $M_\odot$

The differences in the frequencies for the $1.50 M_\odot$ model are somewhat larger than the other two models, with maximum values of up to $0.18\,\mu$Hz in specific cases. This occurs despite the fact that this model is, in general, more smooth than the $1.2 M_\odot$ model. Interestingly, the difference between the variational and Richardson frequencies of ADIPLS are smaller in this case, except for a couple of non-radial modes at the lowest frequencies. Also, the oscillatory pattern observed in the $\nu_{J(V)}$ vs $\nu_{J(R)}$ plots at high frequencies seen in the other two cases is absent in this case.

To conclude, it is encouraging to note that the differences in the frequencies obtained from four independent oscillation codes are on the average about $0.05\,\mu$Hz, and seldom exceed $0.1\,\mu$Hz. This is comparable to the expected best case scenario of observational data. The agreement between the codes is better for the solar model, as might be expected.

On one hand, these comparisons might lead us to improve the numerical techniques involved in the solution of the oscillation equations. On the other, the differences in the frequencies, especially those between the variational and Richardson frequencies, might serve as a test for the structural imperfections in the input stellar models. Similar comparative studies between stellar models generated by different evolution codes might be helpful in this regard. We hope that even better agreement of the frequencies can be obtained by addressing these problems both in the model construction and the solution of oscillation equations.

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